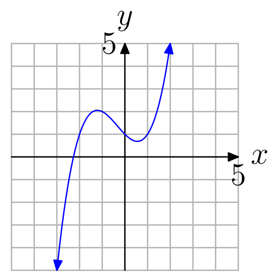
8.4 Exercises

In **Exercises 1-12**, use the graph to determine whether the function is one-to-one.

1.



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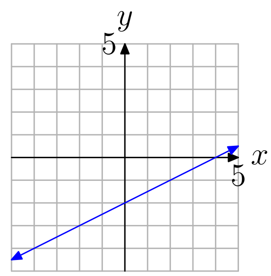

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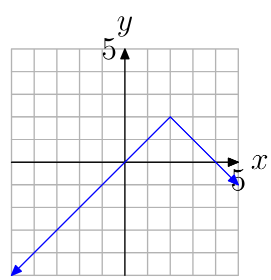

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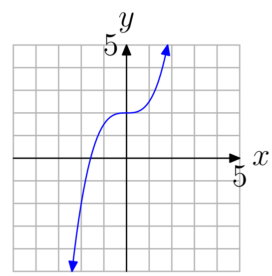

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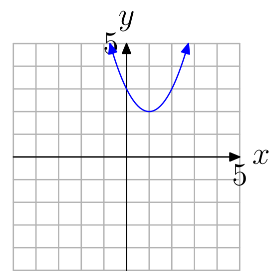

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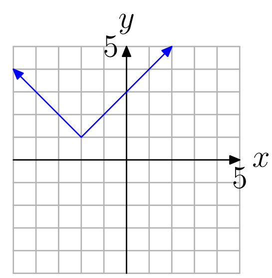

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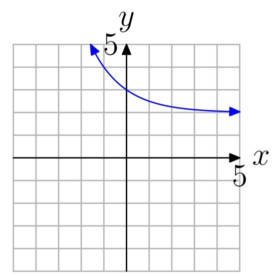

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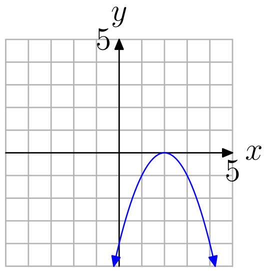

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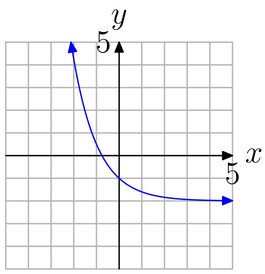

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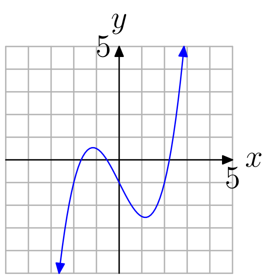

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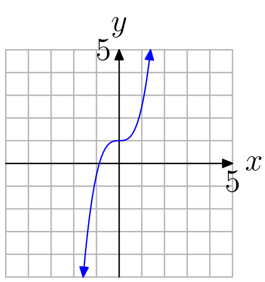

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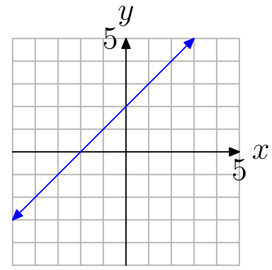


In **Exercises 13-28**, evaluate the composition $g(f(x))$ and simplify your answer.

13. $g(x) = \dfrac{9}{x}$, $f(x) = -2x^2 + 5x - 2$

14. $f(x) = -\dfrac{5}{x}$, $g(x) = -4x^2 + x - 1$

15. $g(x) = 2\sqrt{x}$, $f(x) = -x - 3$

16. $f(x) = 3x^2 - 3x - 5$, $g(x) = \dfrac{6}{x}$

17. $g(x) = 3\sqrt{x}$, $f(x) = 4x + 1$

18. $f(x) = -3x - 5$, $g(x) = -x - 2$

19. $g(x) = -5x^2 + 3x - 4$, $f(x) = \dfrac{5}{x}$

20. $g(x) = 3x + 3$, $f(x) = 4x^2 - 2x - 2$

21. $g(x) = 6\sqrt{x}$, $f(x) = -4x + 4$

22. $g(x) = 5x - 3$, $f(x) = -2x - 4$

23. $g(x) = 3\sqrt{x}$, $f(x) = -2x + 1$

24. $g(x) = \dfrac{3}{x}$, $f(x) = -5x^2 - 5x - 4$

25. $f(x) = \dfrac{5}{x}$, $g(x) = -x + 1$

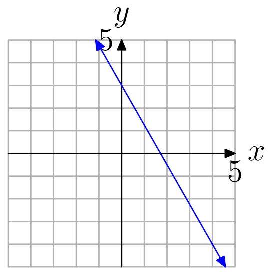
26. $f(x) = 4x^2 + 3x - 4$, $g(x) = \dfrac{2}{x}$

27. $g(x) = -5x + 1,\ f(x) = -3x - 2$

28. $g(x) = 3x^2 + 4x - 3,\ f(x) = \dfrac{8}{x}$

In **Exercises 29-36**, first copy the given graph of the one-to-one function $f(x)$ onto your graph paper. Then on the same coordinate system, sketch the graph of the inverse function $f^{-1}(x)$.

29.



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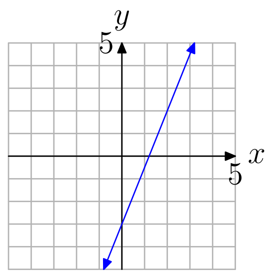

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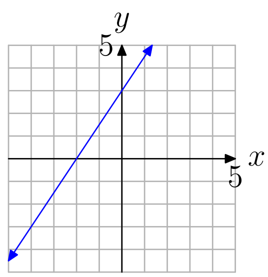

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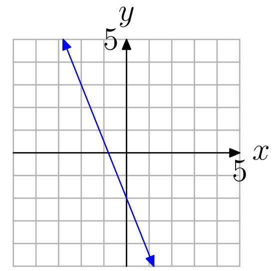

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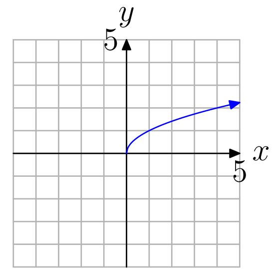

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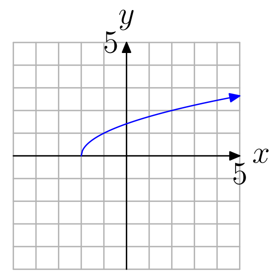

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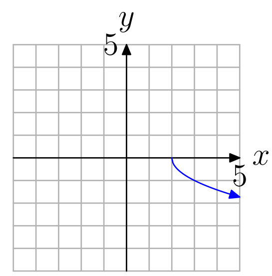

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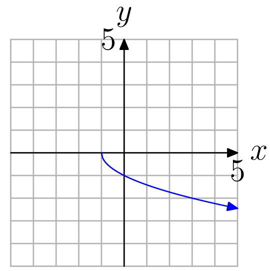


In **Exercises 37-68**, find the formula for the inverse function $f^{-1}(x)$.

37. $f(x) = 5x^3 - 5$

38. $f(x) = 4x^7 - 3$

39. $f(x) = -\dfrac{9x-3}{7x+6}$

40. $f(x) = 6x - 4$

41. $f(x) = 7x - 9$

42. $f(x) = 7x + 4$

43. $f(x) = 3x^5 - 9$

44. $f(x) = 6x + 7$

45. $f(x) = \dfrac{4x+2}{4x+3}$

46. $f(x) = 5x^7 + 4$

47. $f(x) = \dfrac{4x-1}{2x+2}$

48. $f(x) = \sqrt[7]{8x-3}$

49. $f(x) = \sqrt[3]{-6x-4}$

50. $f(x) = \dfrac{8x-7}{3x-6}$

51. $f(x) = \sqrt[7]{-3x-5}$

52. $f(x) = \sqrt[9]{8x+2}$

53. $f(x) = \sqrt[3]{6x+7}$

54. $f(x) = \dfrac{3x+7}{2x+8}$

55. $f(x) = -5x + 2$

56. $f(x) = 6x + 8$

57. $f(x) = 9x^9 + 5$

58. $f(x) = 4x^5 - 9$

59. $f(x) = \dfrac{9x-3}{9x+7}$

60. $f(x) = \sqrt[3]{9x-7}$

61. $f(x) = x^4,\ x \le 0$

62. $f(x) = x^4,\ x \ge 0$

63. $f(x) = x^2 - 1,\ x \le 0$

64. $f(x) = x^2 + 2,\ x \ge 0$

65. $f(x) = x^4 + 3,\ x \le 0$

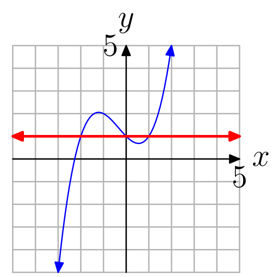
66. $f(x) = x^4 - 5,\ x \ge 0$

67. $f(x) = (x-1)^2,\ x \le 1$

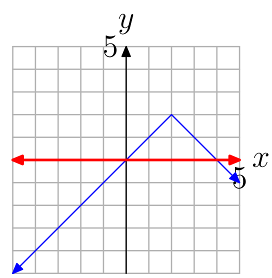
68. $f(x) = (x+2)^2,\ x \ge -2$

8.4 *Solutions*

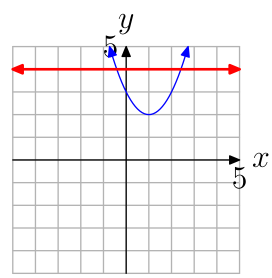
1. The graph fails the horizontal line test. For example, the horizontal line $y = 1$ cuts the graph in more than one place. Therefore, the function not is one-to-one.



3. The graph fails the horizontal line test. For example, the horizontal line $y = 0$ cuts the graph in more than one place. Therefore, the function not is one-to-one.



5. The graph fails the horizontal line test. For example, the horizontal line $y = 4$ cuts the graph in more than one place. Therefore, the function not is one-to-one.



7. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.

9. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.

11. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.

13. Substitute $f(x)$ for x in the expression $\dfrac{9}{x}$ and simplify:

$$g(f(x)) = g(-2x^2 + 5x - 2) = -\frac{9}{2x^2 - 5x + 2}$$

15. Substitute $f(x)$ for x in the expression $2\sqrt{x}$:

$$g(f(x)) = 2\sqrt{-x - 3}$$

17. Substitute $f(x)$ for x in the expression $3\sqrt{x}$:

$$g(f(x)) = 3\sqrt{4x + 1}$$

19. Substitute $f(x)$ for x in the expression $-5x^2 + 3x - 4$ and simplify:

$$g(f(x)) = g\left(\frac{5}{x}\right) = -5\left(\frac{5}{x}\right)^2 + 3\left(\frac{5}{x}\right) - 4 = -\frac{125}{x^2} + \frac{15}{x} - 4$$

21. Substitute $f(x)$ for x in the expression $6\sqrt{x}$:

$$g(f(x)) = 6\sqrt{-4x + 4}$$

23. Substitute $f(x)$ for x in the expression $3\sqrt{x}$:

$$g(f(x)) = 3\sqrt{-2x + 1}$$

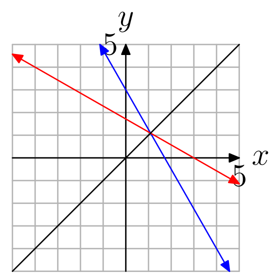
25. Substitute $f(x)$ for x in the expression $-x + 1$ and simplify:

$$g(f(x)) = g(5/x) = -5/x + 1$$

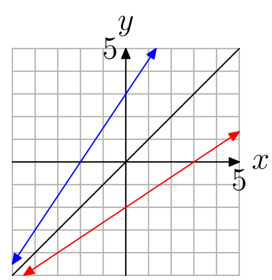
27. Substitute $f(x)$ for x in the expression $-5x + 1$ and simplify:

$$\begin{aligned} g(f(x)) &= g(-3x - 2) \\ &= -5(-3x - 2) + 1 \\ &= 15x + 10 + 1 \\ &= 15x + 11 \end{aligned}$$

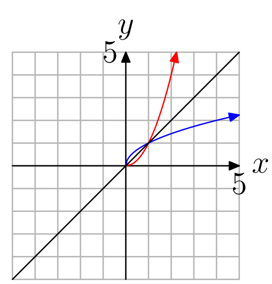
29. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.



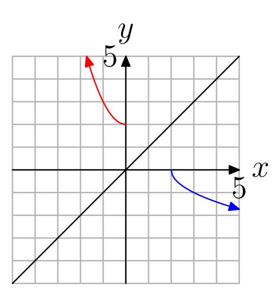
31. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.



33. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.



35. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.



37. Start with the equation $y = 5x^3 - 5$.

Interchange x and y: $x = 5y^3 - 5$.

Then solve for y: $y = \sqrt[3]{\frac{x+5}{5}}$.

39. Start with the equation $y = -\dfrac{9x-3}{7x+6}$.

Interchange x and y: $x = -\dfrac{9y-3}{7y+6}$.

Then solve for y:

$$x(7y+6) = -9y+3 \implies (7x+9)y = -6x+3 \implies y = -\frac{6x-3}{7x+9}$$

41. Start with the equation $y = 7x - 9$.

Interchange x and y: $x = 7y - 9$.

Then solve for y: $y = \frac{x+9}{7}$.

43. Start with the equation $y = 3x^5 - 9$.

Interchange x and y: $x = 3y^5 - 9$.

Then solve for y: $y = \sqrt[5]{\frac{x+9}{3}}$.

45. Start with the equation $y = \dfrac{4x+2}{4x+3}$.

Interchange x and y: $x = \dfrac{4y+2}{4y+3}$.

Then solve for y:

$$x(4y+3) = 4y+2 \implies (4x-4)y = -3x+2 \implies y = -\frac{3x-2}{4x-4}$$

47. Start with the equation $y = \dfrac{4x-1}{2x+2}$.

Interchange x and y: $x = \dfrac{4y-1}{2y+2}$.

Then solve for y:

$$x(2y+2) = 4y-1 \implies (2x-4)y = -2x-1 \implies y = -\frac{2x+1}{2x-4}$$

49. Start with the equation $y = \sqrt[3]{-6x-4}$.

Interchange x and y: $x = \sqrt[3]{-6y-4}$.

Then solve for y: $y = -\frac{x^3+4}{6}$.

51. Start with the equation $y = \sqrt[7]{-3x - 5}$.

Interchange x and y: $x = \sqrt[7]{-3y - 5}$.

Then solve for y: $y = -\frac{x^7+5}{3}$.

53. Start with the equation $y = \sqrt[3]{6x + 7}$.

Interchange x and y: $x = \sqrt[3]{6y + 7}$.

Then solve for y: $y = \frac{x^3-7}{6}$.

55. Start with the equation $y = -5x + 2$.

Interchange x and y: $x = -5y + 2$.

Then solve for y: $y = -\frac{x-2}{5}$.

57. Start with the equation $y = 9x^9 + 5$.

Interchange x and y: $x = 9y^9 + 5$.

Then solve for y: $y = \sqrt[9]{\frac{x-5}{9}}$.

59. Start with the equation $y = \dfrac{9x - 3}{9x + 7}$.

Interchange x and y: $x = \dfrac{9y - 3}{9y + 7}$.

Then solve for y:

$$x(9y + 7) = 9y - 3 \implies (9x - 9)y = -7x - 3 \implies y = -\frac{7x + 3}{9x - 9}$$

61. Start with the equation $y = x^4$ with the domain condition $x \le 0$.

Interchange x and y: $x = y^4$, $y \le 0$.

Solve for y: $y = \pm\sqrt[4]{x}$, $y \le 0$.

The condition $y \le 0$ then implies that $y = -\sqrt[4]{x}$.

63. Start with the equation $y = x^2 - 1$ with the domain condition $x \le 0$.

Interchange x and y: $x = y^2 - 1$, $y \le 0$.

Solve for y: $y = \pm\sqrt{x + 1}$, $y \le 0$.

The condition $y \le 0$ then implies that $y = -\sqrt{x + 1}$.

65. Start with the equation $y = x^4 + 3$ with the domain condition $x \le 0$.

Interchange x and y: $x = y^4 + 3$, $y \le 0$.

Solve for y: $y = \pm\sqrt[4]{x - 3}$, $y \le 0$.

The condition $y \le 0$ then implies that $y = -\sqrt[4]{x - 3}$.

67. Start with the equation $y = (x-1)^2$ with the domain condition $x \leq 1$.
Interchange x and y: $x = (y-1)^2$, $y \leq 1$.
Solve for y: $y = \pm\sqrt{x} + 1$, $y \leq 1$.
The condition $y \leq 1$ then implies that $y = -\sqrt{x} + 1$.