7.1 Exercises

In **Exercises 1-14**, perform each of the following tasks for the given rational function.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis.
ii. Use geometric transformations as in Examples 10, 12, and 13 to draw the graphs of each of the following rational functions. Draw the vertical and horizontal asymptotes as dashed lines and label each with its equation. You may use your calculator to **check** your solution, but you should be able to draw the rational function without the use of a calculator.
iii. Use set-builder notation to describe the domain and range of the given rational function.

1. $f(x) = -2/x$

2. $f(x) = 3/x$

3. $f(x) = 1/(x-4)$

4. $f(x) = 1/(x+3)$

5. $f(x) = 2/(x-5)$

6. $f(x) = -3/(x+6)$

7. $f(x) = 1/x - 2$

8. $f(x) = -1/x + 4$

9. $f(x) = -2/x - 5$

10. $f(x) = 3/x - 5$

11. $f(x) = 1/(x-2) - 3$

12. $f(x) = -1/(x+1) + 5$

13. $f(x) = -2/(x-3) - 4$

14. $f(x) = 3/(x+5) - 2$

In **Exercises 15-22**, find all vertical asymptotes, if any, of the graph of the given function.

15. $f(x) = -\dfrac{5}{x+1} - 3$

16. $f(x) = \dfrac{6}{x+8} + 2$

17. $f(x) = -\dfrac{9}{x+2} - 6$

18. $f(x) = -\dfrac{8}{x-4} - 5$

19. $f(x) = \dfrac{2}{x+5} + 1$

20. $f(x) = -\dfrac{3}{x+9} + 2$

21. $f(x) = \dfrac{7}{x+8} - 9$

22. $f(x) = \dfrac{6}{x-5} - 8$

In **Exercises 23-30**, find all horizontal asymptotes, if any, of the graph of the given function.

23. $f(x) = \dfrac{5}{x+7} + 9$

24. $f(x) = -\dfrac{8}{x+7} - 4$

25. $f(x) = \frac{8}{x+5} - 1$

26. $f(x) = -\frac{2}{x+3} + 8$

27. $f(x) = \frac{7}{x+1} - 9$

28. $f(x) = -\frac{2}{x-1} + 5$

29. $f(x) = \frac{5}{x+2} - 4$

30. $f(x) = -\frac{6}{x-1} - 2$

In **Exercises 31-38**, state the domain of the given rational function using set-builder notation.

31. $f(x) = \frac{4}{x+5} + 5$

32. $f(x) = -\frac{7}{x-6} + 1$

33. $f(x) = \frac{6}{x-5} + 1$

34. $f(x) = -\frac{5}{x-3} - 9$

35. $f(x) = \frac{1}{x+7} + 2$

36. $f(x) = -\frac{2}{x-5} + 4$

37. $f(x) = -\frac{4}{x+2} + 2$

38. $f(x) = \frac{2}{x+6} + 9$

In **Exercises 39-46**, find the range of the given function, and express your answer in set notation.

39. $f(x) = \frac{2}{x-3} + 8$

40. $f(x) = \frac{4}{x-3} + 5$

41. $f(x) = -\frac{5}{x-8} - 5$

42. $f(x) = -\frac{2}{x+1} + 6$

43. $f(x) = \frac{7}{x+7} + 5$

44. $f(x) = -\frac{8}{x+3} + 9$

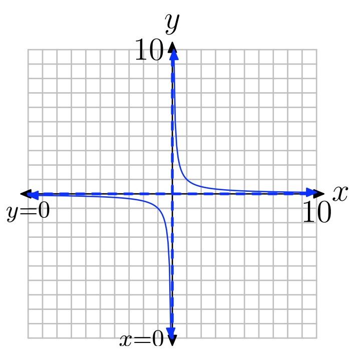
45. $f(x) = \frac{4}{x+3} - 2$

46. $f(x) = -\frac{5}{x-4} + 9$

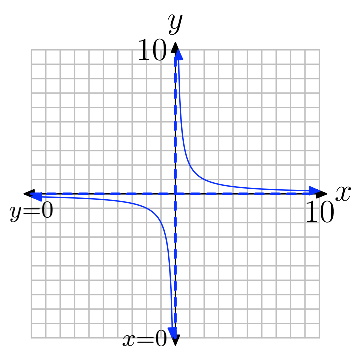
7.1 *Solutions*

1. Start with the graph of $y = 1/x$ in (a). Multiplying by 2 produces the equation $y = 2/x$ and stretches the graph vertically by a factor of 2 as shown in (b). Multiplying by -1 produces the equation $y = -2/x$ and reflects the graph of $y = 2/x$ in (b) over the x-axis to produce the graph of $y = -2/x$ in (c).

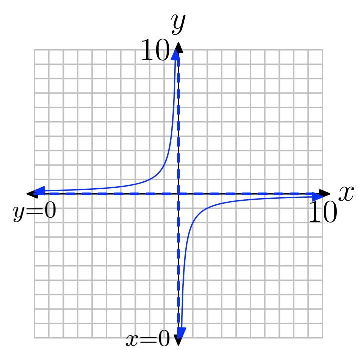
Projecting all the points of the graph in (c) onto the x-axis provides the domain: $D = \{x : x \neq 0\}$. Projecting all the points on the graph in (c) onto the y-axis produces the range: $R = \{y : y \neq 0\}$.



(a) $y = 1/x$.



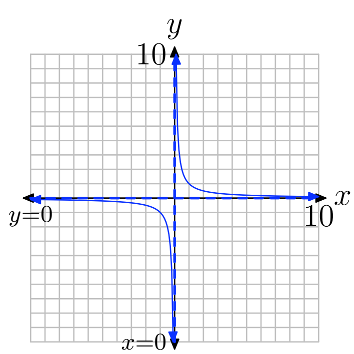
(b) $y = 2/x$.



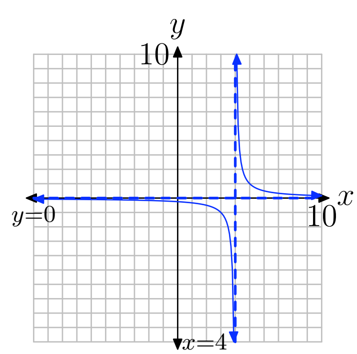
(c) $y = -2/x$.

3. Start with the graph of $y = 1/x$ in (a). Replacing x with $x - 4$ produces the equation $y = 1/(x - 4)$ and slides the graph of $y = 1/x$ four units to the right to produce the graph of $y = 1/(x - 4)$ in (b)

Projecting all the points of the graph in (b) onto the x-axis provides the domain: $D = \{x : x \neq 4\}$. Projecting all the points on the graph in (b) onto the y-axis produces the range: $R = \{y : y \neq 0\}$.



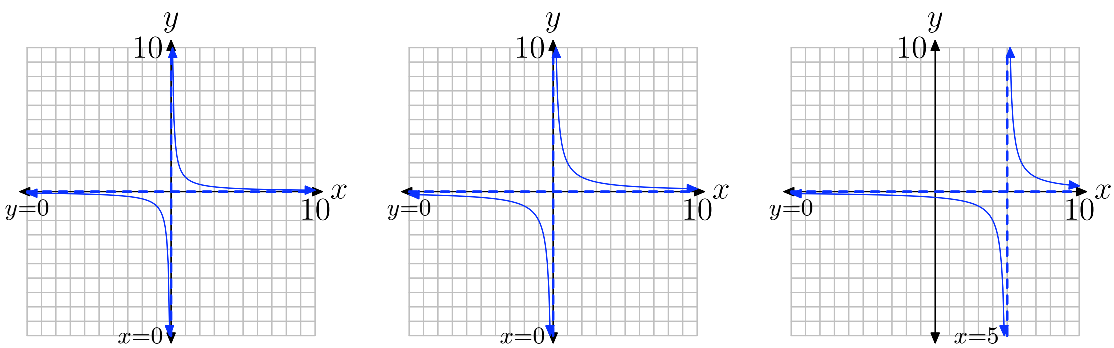
(a) $y = 1/x$.



(b) $y = 1/(x - 4)$.

5. Start with the graph of $y = 1/x$ in (a). Multiply by 2 to produce the equation $y = 2/x$, which stretches the graph of $y = 1/x$ vertically by a factor of 2. The graph of $y = 2/x$ is shown in (b). Finally, replacing x with $x - 5$ produces the equation $y = 2/(x - 5)$ and slides the graph of $y = 2/x$ five units to the right to produce the graph of $y = 2/(x - 5)$ in (c).

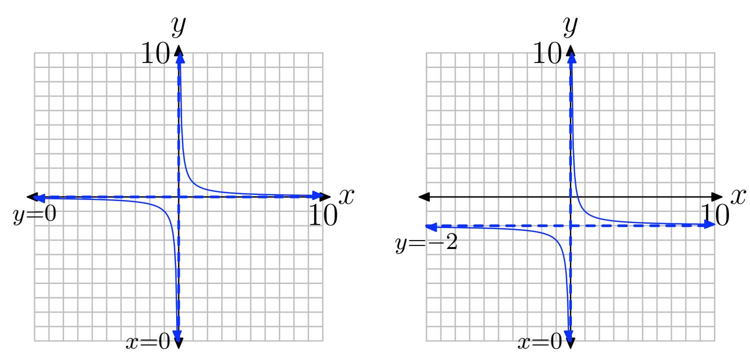
Projecting all the points of the graph in (c) onto the x-axis provides the domain: $D = \{x : x \neq 5\}$. Projecting all the points on the graph in (c) onto the y-axis produces the range: $R = \{y : y \neq 0\}$.



(a) $y = 1/x$. (b) $y = 2/x$. (c) $y = 2/(x - 5)$.

7. Start with the graph of $y = 1/x$ in (a). Subtract 2 to produce the equation $y = 1/x - 2$. This shifts the graph downward 2 units as shown in (b).

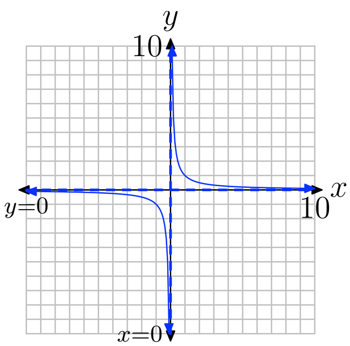
Projecting all the points of the graph in (b) onto the x-axis provides the domain: $D = \{x : x \neq 0\}$. Projecting all the points on the graph in (b) onto the y-axis produces the range: $R = \{y : y \neq -2\}$.



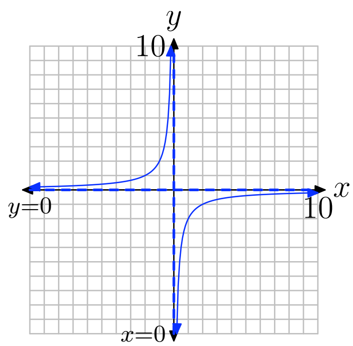
(a) $y = 1/x$. (b) $y = 1/x - 2$.

9. Start with the graph of $y = 1/x$ in (a). Multiply by -2 to produce the equation $y = -2/x$. This stretches the graph of $y = 1/x$ vertically by a factor of 2, then reflects the graph across the x-axis as shown in (b). Subtract 5 to produce the equqtion $y = -2/x - 5$. This shifts the graph of $y = -2/x$ downward 5 units as shown in (c).

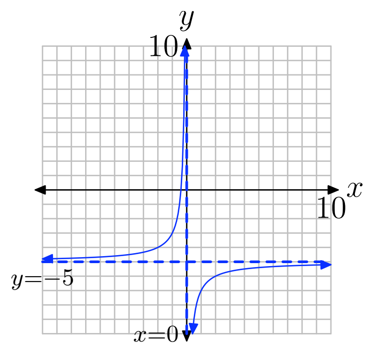
Projecting all the points of the graph in (c) onto the x-axis provides the domain: $D = \{x : x \neq 0\}$. Projecting all the points on the graph in (c) onto the y-axis produces the range: $R = \{y : y \neq -5\}$.



(a) $y = 1/x$.



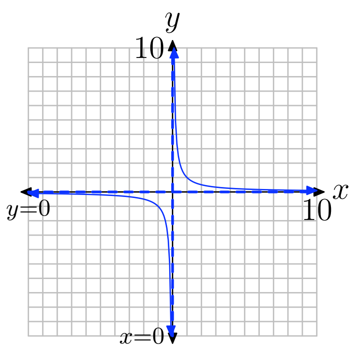
(b) $y = -2/x$.



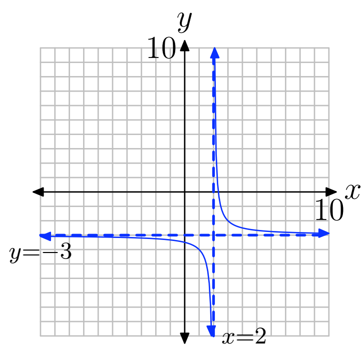
(c) $y = -2/x - 5$.

11. Start with the graph of $y = 1/x$ in (a). Replace x with $x - 2$, then subtract 3 to produce the equation $y = 1/(x - 2) - 3$. This will shift the graph 2 units to the right and 3 units downward, as shown in (b).

Projecting all the points of the graph in (b) onto the x-axis provides the domain: $D = \{x : x \neq 2\}$. Projecting all the points on the graph in (b) onto the y-axis produces the range: $R = \{y : y \neq -3\}$.



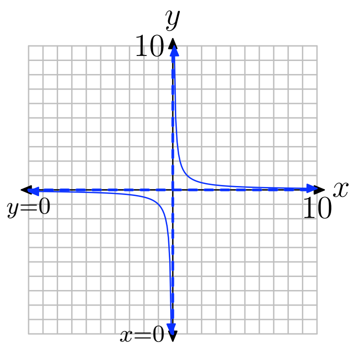
(a) $y = 1/x$.



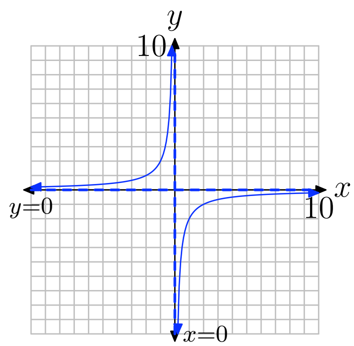
(b) $y = 1/(x - 2) - 3$.

13. Start with the graph of $y = 1/x$ in (a). Multiply by -2 to produce the equation $y = -2/x$. This stretches the graph vertically by a factor of 2 then reflects the graph across the x-axis, as shown in (b). Replace x with $x - 3$, then subtract 4 to produce the equation $y = -2/(x-3) - 4$. This will shift the graph of $y = -2/x$ three units to the right and 4 units downward, as shown in (c).

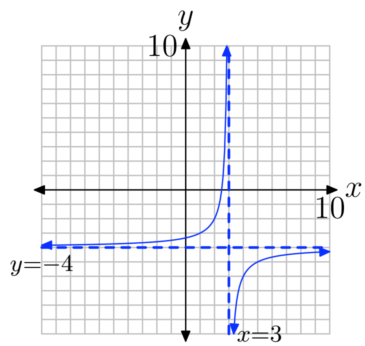
Projecting all the points of the graph in (c) onto the x-axis provides the domain: $D = \{x : x \neq 3\}$. Projecting all the points on the graph in (c) onto the y-axis produces the range: $R = \{y : y \neq -4\}$.



(a) $y = 1/x$.



(b) $y = -2/x$.



(c) $y = -2/(x-3) - 4$.

15. The graph of $f(x) = \frac{5}{x+1} - 3$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 1 unit, (2) a vertical stretch by a factor of 5, (3) a reflection about the x-axis, and (4) a shift down of 3 units. Thus, the vertical asymptote $x = 0$ of the graph of $g(x)$ will also shift left 1 unit to form the vertical asymptote $x = -1$ of the graph of $f(x)$.

17. The graph of $f(x) = \frac{9}{x+2} - 6$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 2 units, (2) a vertical stretch by a factor of 9, (3) a reflection about the x-axis, and (4) a shift down of 6 units. Thus, the vertical asymptote $x = 0$ of the graph of $g(x)$ will also shift left 2 units to form the vertical asymptote $x = -2$ of the graph of $f(x)$.

19. The graph of $f(x) = \frac{2}{x+5} + 1$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 5 units, (2) a vertical stretch by a factor of 2, and (3) a shift up of 1 unit. Thus, the vertical asymptote $x = 0$ of the graph of $g(x)$ will also shift left 5 units to form the vertical asymptote $x = -5$ of the graph of $f(x)$.

21. The graph of $f(x) = \frac{7}{x+8} - 9$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 8 units, (2) a vertical stretch by a factor of 7, and (3) a shift down of 9 units. Thus, the vertical asymptote $x = 0$ of the graph of $g(x)$ will also shift left 8 units to form the vertical asymptote $x = -8$ of the graph of $f(x)$.

23. The graph of $f(x) = \frac{5}{x+7} + 9$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 7 units, (2) a vertical stretch by a factor of 5, and (3) a shift up of 9 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift up 9 units to form the horizontal asymptote $y = 9$ of the graph of $f(x)$.

25. The graph of $f(x) = \frac{8}{x+5} - 1$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 5 units, (2) a vertical stretch by a factor of 8, and (3) a shift down of 1 unit. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift down 1 unit to form the horizontal asymptote $y = -1$ of the graph of $f(x)$.

27. The graph of $f(x) = \frac{7}{x+1} - 9$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 1 unit, (2) a vertical stretch by a factor of 7, and (3) a shift down of 9 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift down 9 units to form the horizontal asymptote $y = -9$ of the graph of $f(x)$.

29. The graph of $f(x) = \frac{5}{x+2} - 4$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 2 units, (2) a vertical stretch by a factor of 5, and (3) a shift down of 4 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift down 4 units to form the horizontal asymptote $y = -4$ of the graph of $f(x)$.

31. An input of $x = -5$ would cause division by zero. Therefore, -5 is not in the domain. All other possible inputs are valid.

33. An input of $x = 5$ would cause division by zero. Therefore, 5 is not in the domain. All other possible inputs are valid.

35. An input of $x = -7$ would cause division by zero. Therefore, -7 is not in the domain. All other possible inputs are valid.

37. An input of $x = -2$ would cause division by zero. Therefore, -2 is not in the domain. All other possible inputs are valid.

39. The graph of $f(x) = \frac{2}{x-3} + 8$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift right of 3 units, (2) a vertical stretch by a factor of 2, and (3) a shift up of 8 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift up 8 units to form the horizontal asymptote $y = 8$ of the graph of $f(x)$. Similarly, $\text{Range}(g) = \{y : y \neq 0\}$ will shift vertically to form $\text{Range}(f) = \{y : y \neq 8\}$. The values of the function will become close to 8 as $x \to \pm\infty$, but never equal to 8.

41. The graph of $f(x) = \frac{5}{x-8} - 5$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift right of 8 units, (2) a vertical stretch by a factor of 5, (3) a reflection about the x-axis, and (4) a shift down of 5 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift down 5 units to form the horizontal asymptote $y = -5$ of the graph of $f(x)$. Similarly, $\text{Range}(g) = \{y : y \neq 0\}$ will shift vertically to form $\text{Range}(f) = \{y : y \neq -5\}$. The values of the function will become close to -5 as $x \to \pm\infty$, but never equal to -5.

43. The graph of $f(x) = \frac{7}{x+7} + 5$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 7 units, (2) a vertical stretch by a factor of 7, and (3) a shift up of 5 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift up 5 units to form the horizontal asymptote $y = 5$ of the graph of $f(x)$. Similarly, $\text{Range}(g) = \{y : y \neq 0\}$ will shift vertically to form $\text{Range}(f) = \{y : y \neq 5\}$. The values of the function will become close to 5 as $x \to \pm\infty$, but never equal to 5.

45. The graph of $f(x) = \frac{4}{x+3} - 2$ can be obtained from the graph of $g(x) = \frac{1}{x}$ by (1) a shift left of 3 units, (2) a vertical stretch by a factor of 4, and (3) a shift down of 2 units. Thus, the horizontal asymptote $y = 0$ of the graph of $g(x)$ will also shift down 2 units to form the horizontal asymptote $y = -2$ of the graph of $f(x)$. Similarly, $\text{Range}(g) = \{y : y \neq 0\}$ will shift vertically to form $\text{Range}(f) = \{y : y \neq -2\}$. The values of the function will become close to -2 as $x \to \pm\infty$, but never equal to -2.

7.2 Exercises

In **Exercises 1-12**, reduce each rational number to lowest terms by applying the following steps:

i. Prime factor both numerator and denominator.
ii. Cancel common prime factors.
iii. Simplify the numerator and denominator of the result.

1. $\dfrac{147}{98}$

2. $\dfrac{3087}{245}$

3. $\dfrac{1715}{196}$

4. $\dfrac{225}{50}$

5. $\dfrac{1715}{441}$

6. $\dfrac{56}{24}$

7. $\dfrac{108}{189}$

8. $\dfrac{75}{500}$

9. $\dfrac{100}{28}$

10. $\dfrac{98}{147}$

11. $\dfrac{1125}{175}$

12. $\dfrac{3087}{8575}$

In **Exercises 13-18**, reduce the given expression to lowest terms. State all restrictions.

13. $\dfrac{x^2-10x+9}{5x-5}$

14. $\dfrac{x^2-9x+20}{x^2-x-20}$

15. $\dfrac{x^2-2x-35}{x^2-7x}$

16. $\dfrac{x^2-15x+54}{x^2+7x-8}$

17. $\dfrac{x^2+2x-63}{x^2+13x+42}$

18. $\dfrac{x^2+13x+42}{9x+63}$

In **Exercises 19-24**, negate any two parts of the fraction, then factor (if necessary) and cancel common factors to reduce the rational expression to lowest terms. State all restrictions.

19. $\dfrac{x+2}{-x-2}$

20. $\dfrac{4-x}{x-4}$

21. $\dfrac{2x-6}{3-x}$

22. $\dfrac{3x+12}{-x-4}$

23. $\dfrac{3x^2+6x}{-x-2}$

24. $\dfrac{8x - 2x^2}{x - 4}$

In **Exercises 25-38**, reduce each of the given rational expressions to lowest terms. State all restrictions.

25. $\dfrac{x^2 - x - 20}{25 - x^2}$

26. $\dfrac{x - x^2}{x^2 - 3x + 2}$

27. $\dfrac{x^2 + 3x - 28}{x^2 + 5x - 36}$

28. $\dfrac{x^2 + 10x + 9}{x^2 + 15x + 54}$

29. $\dfrac{x^2 - x - 56}{8x - x^2}$

30. $\dfrac{x^2 - 7x + 10}{5x - x^2}$

31. $\dfrac{x^2 + 13x + 42}{x^2 - 2x - 63}$

32. $\dfrac{x^2 - 16}{x^2 - x - 12}$

33. $\dfrac{x^2 - 9x + 14}{49 - x^2}$

34. $\dfrac{x^2 + 7x + 12}{9 - x^2}$

35. $\dfrac{x^2 - 3x - 18}{x^2 - 6x + 5}$

36. $\dfrac{x^2 + 5x - 6}{x^2 - 1}$

37. $\dfrac{x^2 - 3x - 10}{-9x - 18}$

38. $\dfrac{x^2 - 6x + 8}{16 - x^2}$

In **Exercises 39-42**, reduce each rational function to lowest terms, and then perform each of the following tasks.

i. Load the original rational expression into `Y1` and the reduced rational expression (your answer) into `Y2` of your graphing calculator.
ii. In `TABLE SETUP`, set `TblStart` equal to zero, Δ`Tbl` equal to 1, then make sure both independent and dependent variables are set to `Auto`. Select `TABLE` and scroll with the up- and down-arrows on your calculator until the smallest restriction is in view. Copy both columns of the table onto your homework paper, showing the agreement between `Y1` and `Y2` and what happens at all restrictions.

39. $\dfrac{x^2 - 8x + 7}{x^2 - 11x + 28}$

40. $\dfrac{x^2 - 5x}{x^2 - 9x}$

41. $\dfrac{8x - x^2}{x^2 - x - 56}$

42. $\dfrac{x^2 + 13x + 40}{-2x - 16}$

Given $f(x) = 2x + 5$, simplify each of the expressions in **Exercises 43-46**. Be sure to reduce your answer to lowest terms and state any restrictions.

43. $\dfrac{f(x) - f(3)}{x - 3}$

44. $\dfrac{f(x) - f(6)}{x - 6}$

45. $\dfrac{f(x) - f(a)}{x - a}$

46. $\dfrac{f(a + h) - f(a)}{h}$

Given $f(x) = x^2 + 2x$, simplify each of the expressions in **Exercises 47-50**. Be sure to reduce your answer to lowest terms and state any restrictions.

47. $\dfrac{f(x) - f(1)}{x - 1}$

48. $\dfrac{f(x) - f(a)}{x - a}$

49. $\dfrac{f(a + h) - f(a)}{h}$

50. $\dfrac{f(x + h) - f(x)}{h}$

Drill for Skill. In **Exercises 51-54**, evaluate the given function at the given expression and simplify your answer.

51. Suppose that f is the function

$$f(x) = -\frac{x - 6}{8x + 7}.$$

Evaluate $f(-3x + 2)$ and simplify your answer.

52. Suppose that f is the function

$$f(x) = -\frac{5x + 3}{7x + 6}.$$

Evaluate $f(-5x + 1)$ and simplify your answer.

53. Suppose that f is the function

$$f(x) = -\frac{3x - 6}{4x + 6}.$$

Evaluate $f(-x-3)$ and simplify your answer.

54. Suppose that f is the function

$$f(x) = \frac{4x - 1}{2x - 4}.$$

Evaluate $f(5x)$ and simplify your answer.

7.2 *Solutions*

1.

$$\frac{147}{98} = \frac{3 \cdot 7 \cdot 7}{2 \cdot 7 \cdot 7} = \frac{3}{2} = \frac{3}{2}$$

3.

$$\frac{1715}{196} = \frac{5 \cdot 7 \cdot 7 \cdot 7}{2 \cdot 2 \cdot 7 \cdot 7} = \frac{5 \cdot 7}{2 \cdot 2} = \frac{35}{4}$$

5.

$$\frac{1715}{441} = \frac{5 \cdot 7 \cdot 7 \cdot 7}{3 \cdot 3 \cdot 7 \cdot 7} = \frac{5 \cdot 7}{3 \cdot 3} = \frac{35}{9}$$

7.

$$\frac{108}{189} = \frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 7} = \frac{2 \cdot 2}{7} = \frac{4}{7}$$

9.

$$\frac{100}{28} = \frac{2 \cdot 2 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 7} = \frac{5 \cdot 5}{7} = \frac{25}{7}$$

11.

$$\frac{1125}{175} = \frac{3 \cdot 3 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 7} = \frac{3 \cdot 3 \cdot 5}{7} = \frac{45}{7}$$

13.

$$\frac{x^2 - 10x + 9}{5x - 5} = \frac{(x-1)(x-9)}{5(x-1)} = \frac{x-9}{5},$$

provided $x \neq 1$

15.

$$\frac{x^2 - 2x - 35}{x^2 - 7x} = \frac{(x-7)(x+5)}{x(x-7)} = \frac{x+5}{x},$$

provided $x \neq 0$, 7

17.

$$\frac{x^2+2x-63}{x^2+13x+42} = \frac{(x-7)(x+9)}{(x+7)(x+6)},$$

provided $x \neq -7$, -6, and this expression does not simplify any further.

19. Negate the denominator and the fraction bar.

$$\frac{x+2}{-x-2} = -\frac{x+2}{x+2} = -1$$

This result is valid provided $x \neq -2$.

21. Negate the denominator and the fraction bar.

$$\frac{2x-6}{3-x} = -\frac{2x-6}{x-3}$$

Factor, then cancel.

$$-\frac{2x-6}{x-3} = -\frac{2(x-3)}{x-3} = -\frac{2\cancel{(x-3)}}{\cancel{x-3}} = -2$$

This result is valid provided $x \neq 3$.

23. Negate the denominator and the fraction bar.

$$\frac{3x^2+6x}{-x-2} = -\frac{3x^2+6x}{x+2}$$

Factor, then cancel.

$$-\frac{3x^2+6x}{x+2} = -\frac{3x(x+2)}{x+2} = -\frac{3x\cancel{(x+2)}}{\cancel{x+2}} = -3x \tag{1}$$

This result is valid provided $x \neq -2$.

25.

$$\frac{x^2-x-20}{25-x^2} = -\frac{x^2-x-20}{x^2-25} = -\frac{(x-5)(x+4)}{(x-5)(x+5)} = -\frac{x+4}{x+5},$$

provided $x \neq -5$, 5

27.

$$\frac{x^2+3x-28}{x^2+5x-36} = \frac{(x-4)(x+7)}{(x-4)(x+9)} = \frac{x+7}{x+9},$$

provided $x \neq 4$, -9

29.

$$\frac{x^2 - x - 56}{8x - x^2} = -\frac{x^2 - x - 56}{x^2 - 8x} = -\frac{(x-8)(x+7)}{x(x-8)} = -\frac{x+7}{x},$$

provided $x \neq 0$, 8

31.

$$\frac{x^2 + 13x + 42}{x^2 - 2x - 63} = \frac{(x+7)(x+6)}{(x+7)(x-9)} = \frac{x+6}{x-9},$$

provided $x \neq -7$, 9

33.

$$\frac{x^2 - 9x + 14}{49 - x^2} = -\frac{x^2 - 9x + 14}{x^2 - 49} = -\frac{(x-7)(x-2)}{(x-7)(x+7)} = -\frac{x-2}{x+7},$$

provided $x \neq 7$, -7

35.

$$\frac{x^2 - 3x - 18}{x^2 - 6x + 5} = \frac{(x-6)(x+3)}{(x-1)(x-5)},$$

provided $x \neq 1$, 5, and this expression does not simplify any further.

37.

$$\frac{x^2 - 3x - 10}{-9x - 18} = \frac{(x+2)(x-5)}{-9(x+2)} = -\frac{x-5}{9}$$

provided $x \neq -2$

39.

$$\frac{x^2 - 8x + 7}{x^2 - 11x + 28} = \frac{(x-7)(x-1)}{(x-7)(x-4)} = \frac{x-1}{x-4},$$

provided $x \neq 7$, 4

X	Y1	Y2
3	-2	-2
4	Err:	Err:
5	4	4
6	2.5	2.5
7	Err:	2
8	1.75	1.75

41.

$$\frac{8x - x^2}{x^2 - x - 56} = -\frac{x^2 - 8x}{x^2 - x - 56} = -\frac{x(x-8)}{(x-8)(x+7)} = -\frac{x}{x+7},$$

provided $x \neq -7$, 8

X	Y1	Y2
-8	-8	-8
-7	Err:	Err:
-6	6	6
-5	2.5	2.5
-4	1.33333	1.33333
-3	0.75	0.75
-2	0.4	0.4
-1	0.166667	0.166667
0	-0	-0
1	-0.125	-0.125
2	-0.222222	-0.222222
3	-0.3	-0.3
4	-0.363636	-0.363636
5	-0.416667	-0.416667
6	-0.461538	-0.461538
7	-0.5	-0.5
8	Err:	-0.533333
9	-0.5625	-0.5625

43.

$$\begin{aligned}\frac{f(x) - f(3)}{x - 3} &= \frac{(2x+5) - (2(3)+5)}{x-3} \\ &= \frac{2x-6}{x-3} \\ &= \frac{2(x-3)}{x-3} \\ &= 2,\end{aligned}$$

provided $x \neq 3$.

45.

$$\begin{aligned}\frac{f(x)-f(a)}{x-a} &= \frac{(2x+5)-(2(a)+5)}{x-3}\\ &= \frac{2x-2a}{x-a}\\ &= \frac{2(x-a)}{x-a}\\ &= 2,\end{aligned}$$

provided $x \neq a$.

47.

$$\begin{aligned}\frac{f(x)-f(1)}{x-1} &= \frac{(x^2+2x)-((1)^2+2(1))}{x-1}\\ &= \frac{x^2+2x-3}{x-1}\\ &= \frac{(x+3)(x-1)}{x-1}\\ &= x+3,\end{aligned}$$

provided $x \neq 1$.

49.

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{[(a+h)^2+2(a+h)]-[a^2+2a]}{h}\\ &= \frac{a^2+2ah+h^2+2a+2h-a^2-2a}{h}\\ &= \frac{2ah+h^2+2h}{h}\\ &= \frac{h(2a+h+2)}{h}\\ &= 2a+h+2,\end{aligned}$$

provided $h \neq 0$.

51. Substitute $-3x+2$ for x in $-\frac{x-6}{8x+7}$ and simplify to get $-\frac{3x+4}{24x-23}$.

53. Substitute $-x-3$ for x in $-\frac{3x-6}{4x+6}$ and simplify to get $-\frac{3x+15}{4x+6}$.

7.3 Exercises

For rational functions **Exercises 1-20**, follow the Procedure for Graphing Rational Functions in the narrative, performing each of the following tasks.

For rational functions **Exercises 1-20**, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.
ii. Perform each of the nine steps listed in the Procedure for Graphing Rational Functions in the narrative.

1. $f(x) = (x-3)/(x+2)$

2. $f(x) = (x+2)/(x-4)$

3. $f(x) = (5-x)/(x+1)$

4. $f(x) = (x+2)/(4-x)$

5. $f(x) = (2x-5)/(x+1)$

6. $f(x) = (2x+5/(3-x)$

7. $f(x) = (x+2)/(x^2-2x-3)$

8. $f(x) = (x-3)/(x^2-3x-4)$

9. $f(x) = (x+1)/(x^2+x-2)$

10. $f(x) = (x-1)/(x^2-x-2)$

11. $f(x) = (x^2-2x)/(x^2+x-2)$

12. $f(x) = (x^2-2x)/(x^2-2x-8)$

13. $f(x) = (2x^2-2x-4)/(x^2-x-12)$

14. $f(x) = (8x-2x^2)/(x^2-x-6)$

15. $f(x) = (x-3)/(x^2-5x+6)$

16. $f(x) = (2x-4)/(x^2-x-2)$

17. $f(x) = (2x^2-x-6)/(x^2-2x)$

18. $f(x) = (2x^2-x-6)/(x^2-2x)$

19. $f(x) = (4+2x-2x^2)/(x^2+4x+3)$

20. $f(x) = (3x^2-6x-9)/(1-x^2)$

In **Exercises 21-28**, find the coordinates of the x-intercept(s) of the graph of the given rational function.

21. $f(x) = \dfrac{81-x^2}{x^2+10x+9}$

22. $f(x) = \dfrac{x-x^2}{x^2+5x-6}$

23. $f(x) = \dfrac{x^2-x-12}{x^2+2x-3}$

24. $f(x) = \dfrac{x^2-81}{x^2-4x-45}$

25. $f(x) = \dfrac{6x-18}{x^2-7x+12}$

26. $f(x) = \dfrac{4x+36}{x^2+15x+54}$

27. $f(x) = \dfrac{x^2-9x+14}{x^2-2x}$

28. $f(x) = \dfrac{x^2-5x-36}{x^2-9x+20}$

In **Exercises 29-36**, find the equations of all vertical asymptotes.

29. $f(x) = \dfrac{x^2 - 7x}{x^2 - 2x}$

30. $f(x) = \dfrac{x^2 + 4x - 45}{3x + 27}$

31. $f(x) = \dfrac{x^2 - 6x + 8}{x^2 - 16}$

32. $f(x) = \dfrac{x^2 - 11x + 18}{2x - x^2}$

33. $f(x) = \dfrac{x^2 + x - 12}{-4x + 12}$

34. $f(x) = \dfrac{x^2 - 3x - 54}{9x - x^2}$

35. $f(x) = \dfrac{16 - x^2}{x^2 + 7x + 12}$

36. $f(x) = \dfrac{x^2 - 11x + 30}{-8x + 48}$

In **Exercises 37-42**, use a graphing calculator to determine the behavior of the given rational function as x approaches both positive and negative infinity by performing the following tasks:

i. Load the rational function into the `Y=` menu of your calculator.
ii. Use the `TABLE` feature of your calculator to determine the value of $f(x)$ for $x = 10$, 100, 1000, and 10000. Record these results on your homework in table form.
iii. Use the `TABLE` feature of your calculator to determine the value of $f(x)$ for $x = -10$, -100, -1000, and -10000. Record these results on your homework in table form.
iv. Use the results of your tabular exploration to determine the equation of the horizontal asymptote.

37. $f(x) = (2x + 3)/(x - 8)$

38. $f(x) = (4 - 3x)/(x + 2)$

39. $f(x) = (4 - x^2)/(x^2 + 4x + 3)$

40. $f(x) = (10 - 2x^2)/(x^2 - 4)$

41. $f(x) = (x^2 - 2x - 3)/(2x^2 - 3x - 2)$

42. $f(x) = (2x^2 - 3x - 5)/(x^2 - x - 6)$

In **Exercises 43-48**, use a purely analytical method to determine the domain of the given rational function. Describe the domain using set-builder notation.

43. $f(x) = \dfrac{x^2 - 5x - 6}{-9x - 9}$

44. $f(x) = \dfrac{x^2 + 4x + 3}{x^2 - 5x - 6}$

45. $f(x) = \dfrac{x^2 + 5x - 24}{x^2 - 3x}$

46. $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 5x - 6}$

47. $f(x) = \dfrac{x^2 - 4x + 3}{x - x^2}$

48. $f(x) = \dfrac{x^2 - 4}{x^2 - 9x + 14}$

7.3 *Solutions*

1. Step 1: The numerator and denominator of

$$f(x) = \frac{x-3}{x+2}$$

are already factored.

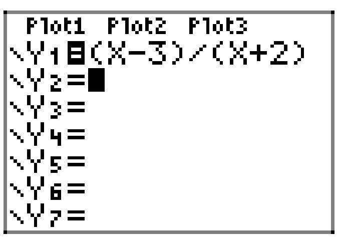
Step 2: Note that $x = -2$ makes the denominator zero and is a restriction.

Step 3: The number $x = 3$ will make the numerator of the rational function $f(x) = (x-3)/(x+2)$ equal to zero without making the denominator equal to zero (it is not a restriction). Hence, $x = 3$ is a zero and $(3, 0)$ will be an x-intercept of the graph of f.

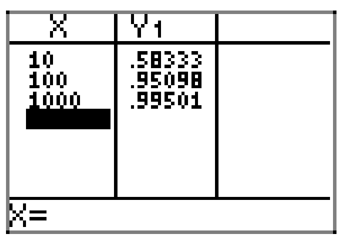
Step 4: Since f is already reduced to lowest terms, we note that the restriction $x = -2$ will place a vertical asymptote in the graph of f with equation $x = -2$.

Step 5: We will calculate and plot two points, one on each side of the vertical asymptote: $(-1, -4)$ and $(-3, 6)$.

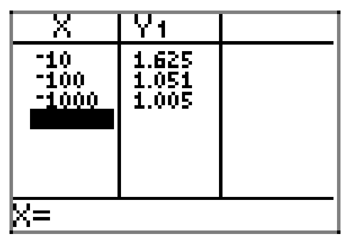
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



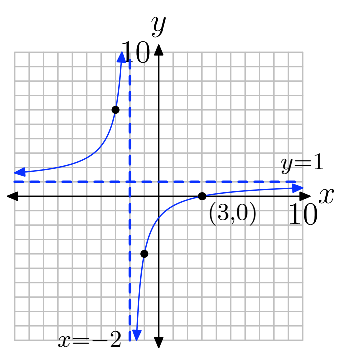
(b)



(c)

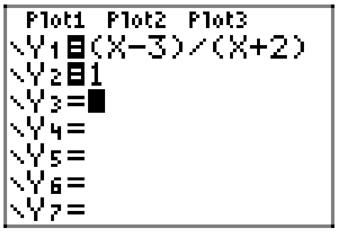
Thus, the line $y = 1$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.

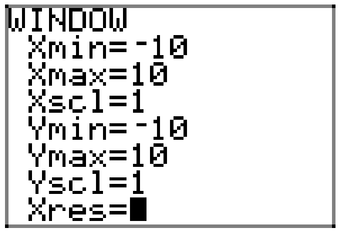


Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

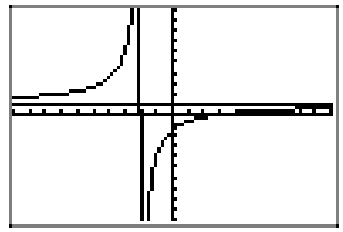
Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a)



(b)



(c)

3. **Step 1:** The numerator and denominator of

$$f(x) = \frac{5 - x}{x + 1}$$

are already factored.

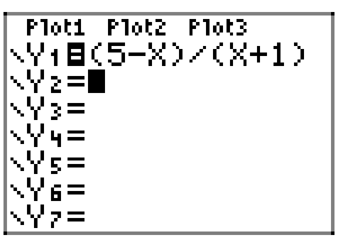
Step 2: Note that $x = -1$ makes the denominator zero and is a restriction.

Step 3: Note that the number $x = 5$ will make the numerator of the rational function $f(x) = (5 - x)/(x + 1)$ equal to zero without making the denominator equal to zero (it is not a restriction). Hence, $x = 5$ is a zero and $(5, 0)$ will be an x-intercept of the graph of f.

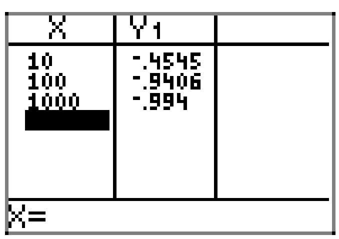
Step 4: Since f is already reduced to lowest terms, we note that the restriction $x = -1$ will place a vertical asymptote in the graph of f with equation $x = -1$.

Step 5: We will calculate and plot two points, one on each side of the vertical asymptote: $(-2, -7)$ and $(0, 5)$.

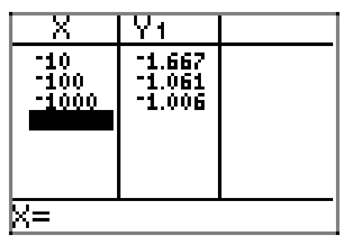
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



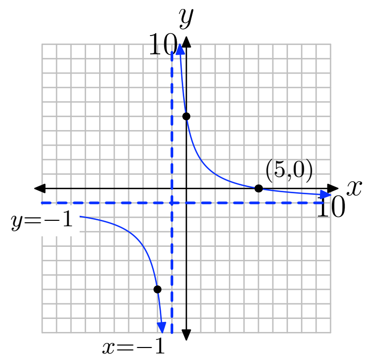
(b)



(c)

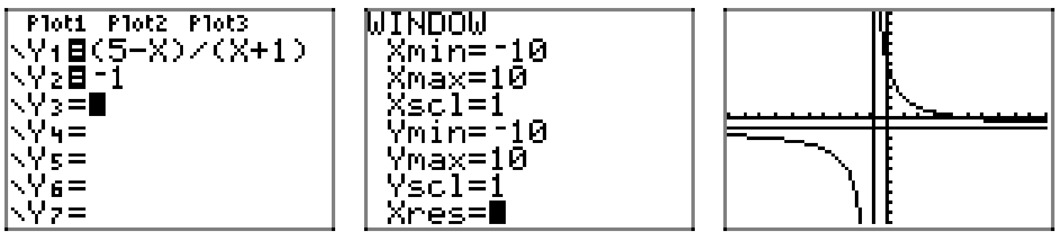
Thus, the line $y = -1$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.



Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a) (b) (c)

5. Step 1: The numerator and denominator of

$$f(x) = \frac{2x-5}{x+1}$$

are already factored.

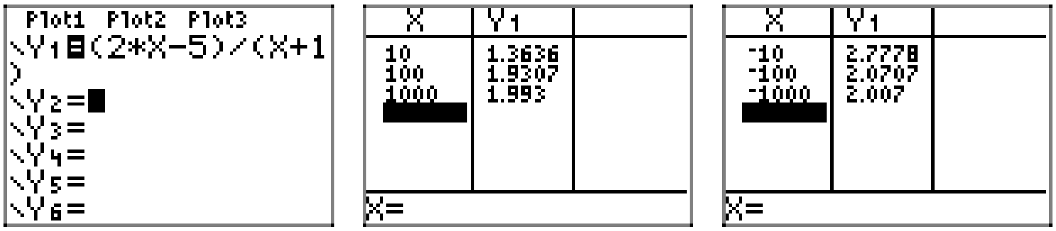
Step 2: Note that $x = -1$ makes the denominator zero and is a restriction.

Step 3: Note that the number $x = 5/2$ will make the numerator of the rational function $f(x) = (2x-5)/(x+1)$ equal to zero without making the denominator equal to zero (it is not a restriction). Hence, $x = 5/2$ is a zero and $(5/2, 0)$ will be an x-intercept of the graph of f.

Step 4: Since f is already reduced to lowest terms, we note that the restriction $x = -1$ will place a vertical asymptote in the graph of f with equation $x = -1$.

Step 5: We will calculate and plot two points, one on each side of the vertical asymptote: $(-2, 9)$ and $(0, -5)$.

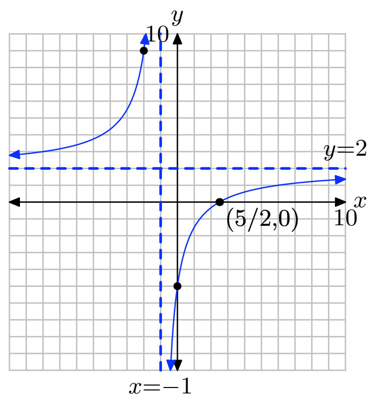
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a) (b) (c)

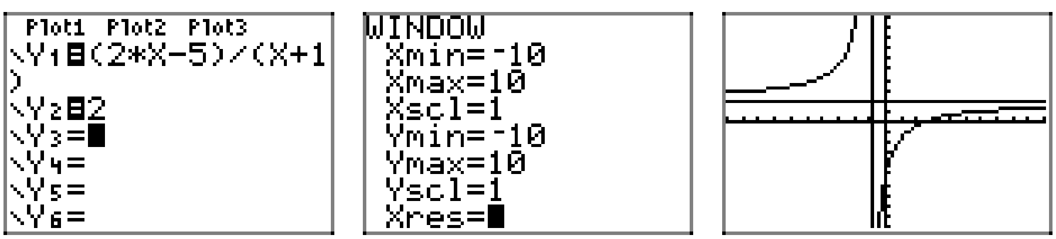
Thus, the line $y = 2$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.



Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a) (b) (c)

7. **Step 1:** Factor numerator and denominator to obtain

$$f(x) = \frac{x+2}{(x+1)(x-3)}.$$

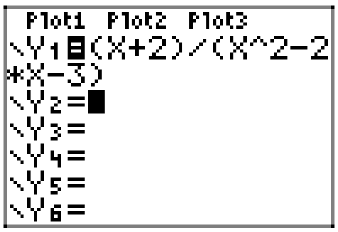
Step 2: Note the restrictions are $x = -1$ and $x = 3$.

Step 3: The number $x = -2$ will make the numerator of the rational function $f(x) = (x+2)/((x+1)(x-3))$ equal to zero without making the denominator equal to zero (it is not a restriction). Hence, $x = -2$ is a zero and $(-2, 0)$ will be an x-intercept of the graph of f.

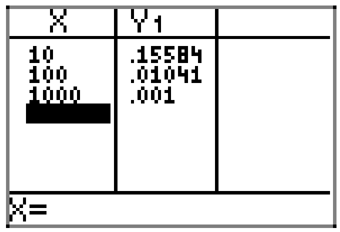
Step 4: Since f is already reduced to lowest terms, we note that the restrictions $x = -1$ and $x = 3$ will place vertical asymptotes in the graph of f with equations $x = -1$ and $x = 3$.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-1.5, 0.2222)$, $(0, -0.6667)$, and $(4, 1.2)$. These points are approximations, not exact.

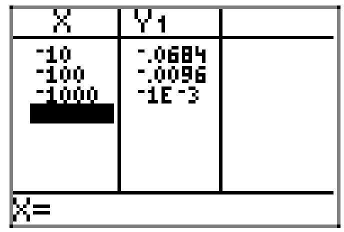
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



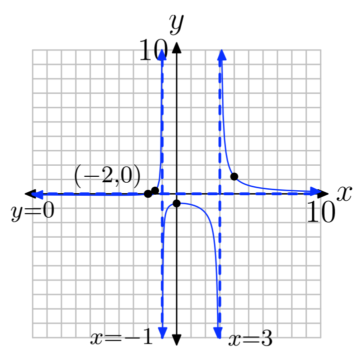
(b)



(c)

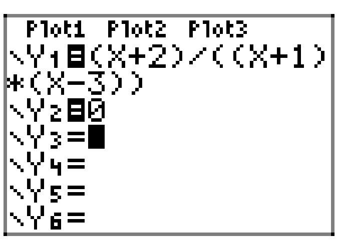
Thus, the line $y = 0$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.

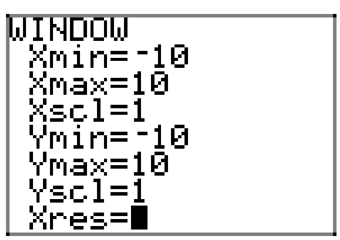


Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

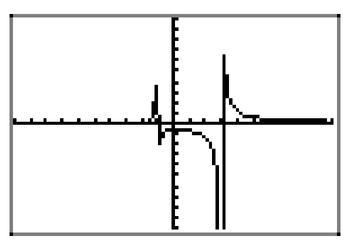
Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images. Note that the image in (c) is not very good, but it is enough to convince us that our work above is reasonable.



(a)



(b)



(c)

9. **Step 1:** Factor numerator and denominator to obtain

$$f(x) = \frac{x+1}{(x+2)(x-1)}.$$

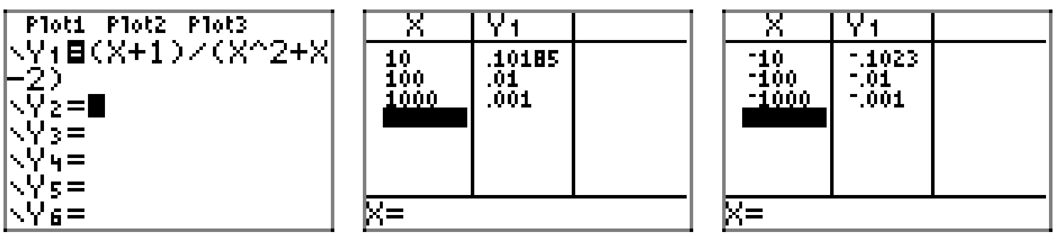
Step 2: The restrictions are $x = -2$ and $x = 1$.

Step 3: The number $x = -1$ will make the numerator of the rational function $f(x) = (x+1)/((x+2)(x-1))$ equal to zero without making the denominator equal to zero (it is not a restriction). Hence, $x = -1$ is a zero and $(-1, 0)$ will be an x-intercept of the graph of f.

Step 4: Since f is already reduced to lowest terms, we note that the restrictions $x = -2$ and $x = 1$ will place vertical asymptotes in the graph of f with equations $x = -2$ and $x = 1$.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-3, -0.5)$, $(0, -0.5)$, and $(2, 0.75)$. These points are approximations, not exact.

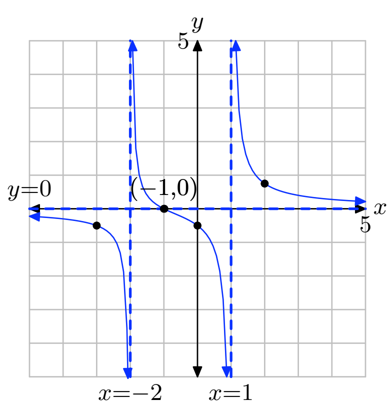
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a) (b) (c)

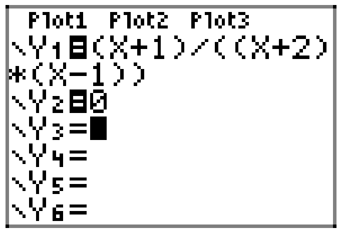
Thus, a horizontal asymptote is the line $y = 0$.

Step 7: Putting all of this information together allows us to draw the following graph.

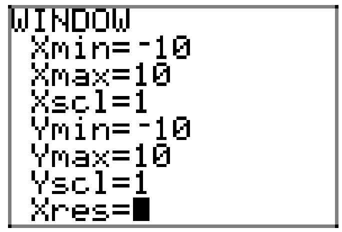


Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

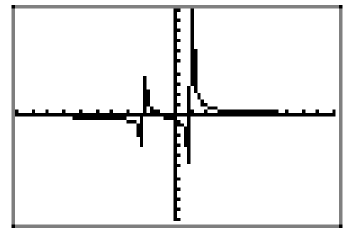
Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images. Note that the image in (c) is not very good, but it is enough to convince us that our work above is reasonable.



(a)



(b)


(c)

11. **Step 1:** Factor numerator and denominator to obtain

$$f(x) = \frac{x(x-2)}{(x+2)(x-1)}.$$

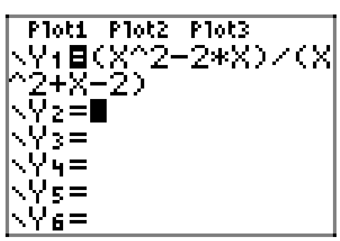
Step 2: The restrictions are $x = -2$ and $x = 1$.

Step 3: The numbers $x = 0$ and $x = 2$ will make the numerator of the rational function $f(x) = x(x-2)/((x+2)(x-1))$ equal to zero without making the denominator equal to zero (they are not restrictions). Hence, $x = 0$ and $x = 2$ are zeros and $(0, 0)$ and $(2, 0)$ will be x-intercepts of the graph of f.

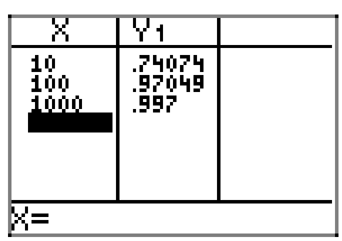
Step 4: Since f is already reduced to lowest terms, we note that the restrictions $x = -2$ and $x = 1$ will place vertical asymptotes in the graph of f with equations $x = -2$ and $x = 1$.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-3, 3.75)$, $(-1, -1.5)$, $(0.5, 0.6)$, and $(1.5, -0.4286)$. These points are approximations, not exact.

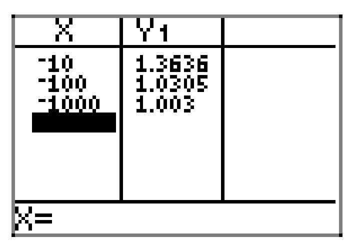
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



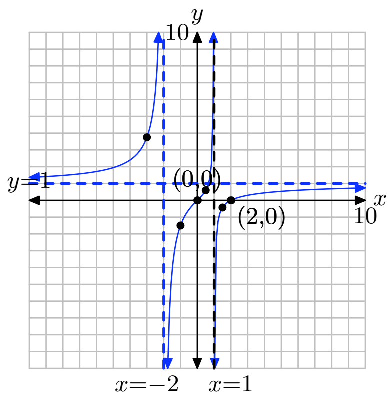
(b)



(c)

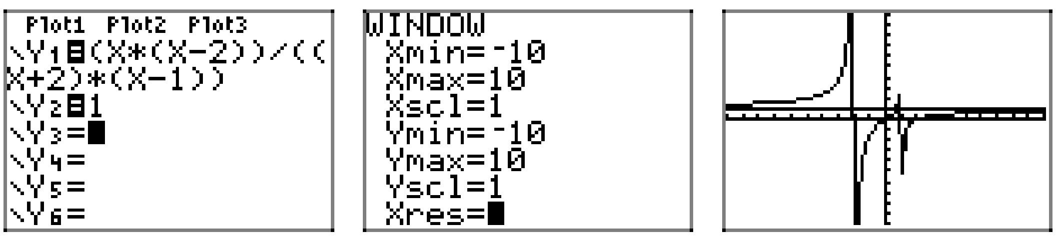
Thus, the line $y = 1$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.



Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images. Note that the image in (c) is not very good, but it is enough to convince us that our work above is reasonable.



(a) (b) (c)

13. **Step 1:** Factor.

$$f(x) = \frac{2x^2 - 2x - 4}{x^2 - x - 12} = \frac{2(x^2 - x - 2)}{x^2 - x - 12} = \frac{2(x-2)(x+1)}{(x-4)(x+3)}$$

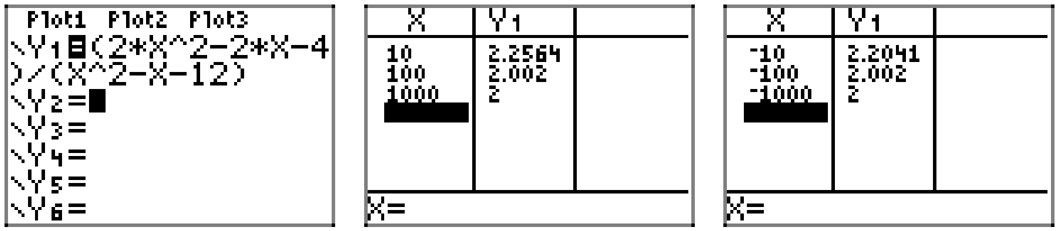
Step 2: The restrictions are $x = 4$ and $x = -3$.

Step 3: The numbers $x = 2$ and $x = -1$ will make the numerator of the rational function f equal to zero without making the denominator equal to zero (they are not restrictions). Hence, $x = 2$ and $x = -1$ are zeros and $(2, 0)$ and $(-1, 0)$ will be x-intercepts of the graph of f.

Step 4: Since f is already reduced to lowest terms, we note that the restrictions $x = 4$ and $x = -3$ will place vertical asymptotes in the graph of f with equations $x = 4$ and $x = -3$.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-4, 4.5)$, $(-2, -1.3333)$, $(3, -1.3333)$, and $(5, 4.5)$. These points are approximations, not exact.

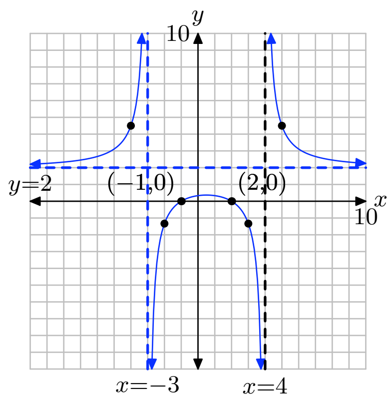
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a) (b) (c)

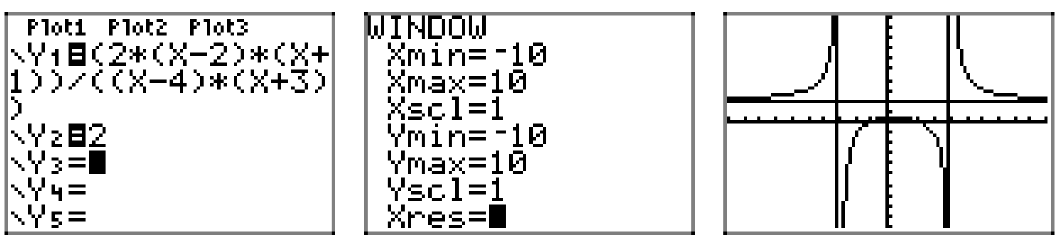
Thus, the line $y = 2$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.



Step 8: The restrictions of the reduced form are the same as the restrictions of the original form. Hence, there are no "holes" in the graph.

Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a) (b) (c)

15. **Step 1:** Factor.

$$f(x) = \frac{x-3}{x^2-5x+6} = \frac{x-3}{(x-3)(x-2)}$$

Step 2: The restrictions are $x = 3$ and $x = 2$.

Step 3: There is no number that will make the numerator zero without making the denominator zero. Hence, the function has no zeros and the graph has no x-intercept.

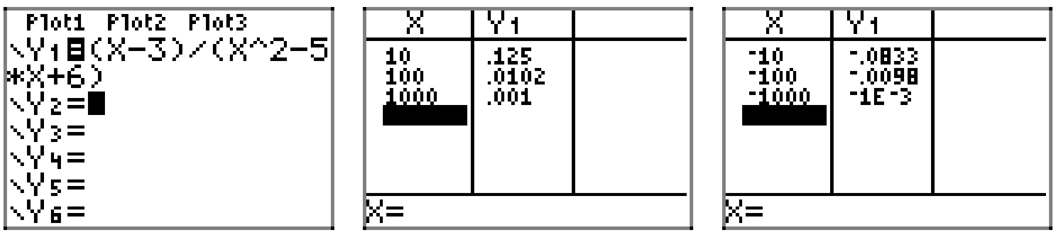
Step 4: Reduce, obtaining the new function

$$g(x) = \frac{1}{x-2}.$$

Note that $x = 2$ is still a restriction of the reduced form, so the graph of f must have vertical asymptote with equation $x = 2$. Note that $x = 3$ is no longer a restriction of the reduced form and will cause a "hole" in the graph, which we will deal with shortly.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(1, -1)$ and $(4, 0.5)$.

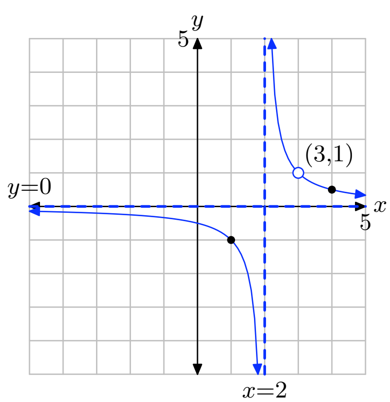
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a) (b) (c)

Thus, the line $y = 0$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.

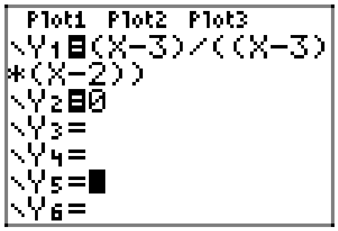


Step 8: Recall that we determined that the graph of f will have a "hole" at the restriction $x = 3$. Use the reduced form $g(x) = 1/(x-2)$ to compute the y-value of the "hole."

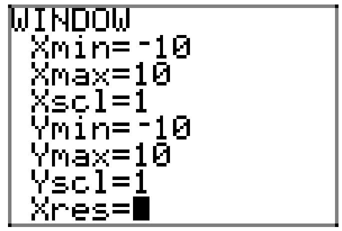
$$g(3) = \frac{1}{3-2} = 1$$

Hence, there will be a "hole" in the graph of f at $(3, 1)$.

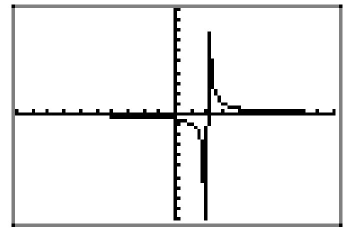
Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images. Note that the image in (c) is not very good, but it is enough to convince us that our work above is reasonable.



(a)



(b)



(c)

17. **Step 1:** Factor,

$$f(x) = \frac{2x^2 - x - 6}{x^2 - 2x} = \frac{(2x+3)(x-2)}{x(x-2)}$$

Step 2: The restrictions are $x = 0$ and $x = 2$.

Step 3: The number $x = -3/2$ makes the numerator zero without making the denominator zero (it is not a restriction). Hence, $x = -3/2$ is a zero of f and $(-3/2, 0)$ is an x-intercept of the graph of f.

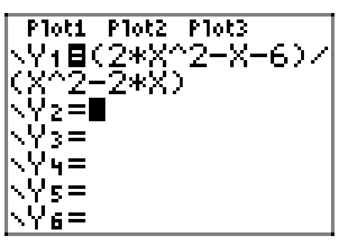
Step 4: Reduce to obtain the new function

$$g(x) = \frac{2x+3}{x}$$

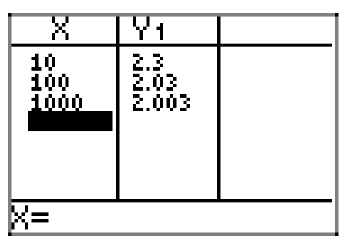
Note that $x = 0$ is still a restriction of the reduced form, so the graph of f must have vertical asymptote with equation $x = 0$. Note that $x = 2$ is no longer a restriction of the reduced form. Hence the graph of f will have a "hole" at this restriction. We will deal with this point in a moment.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-1, -1)$ and $(1, 5)$.

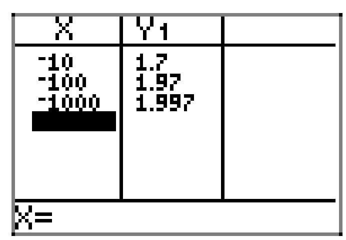
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



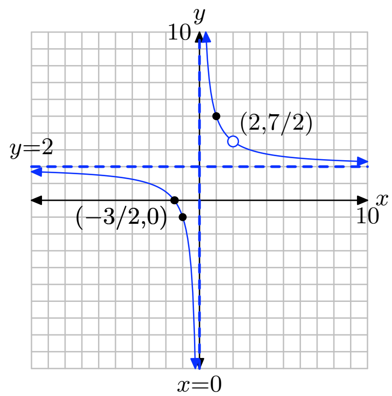
(b)



(c)

Thus, the line $y = 2$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.

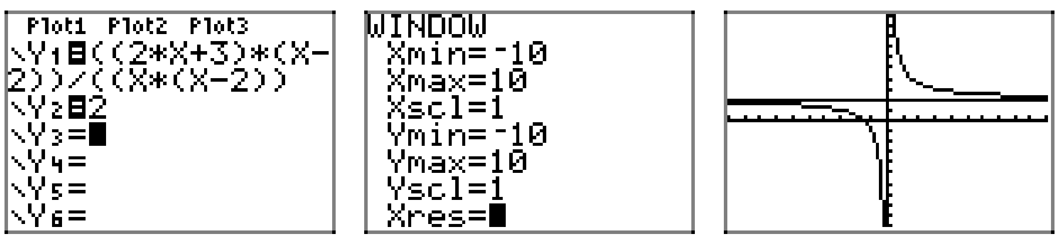


Step 8: Recall that we determined that the graph of f will have a "hole" at the restriction $x = 2$. Use the reduced form $g(x) = (2x + 3)/x$ to determine the y-value of the "hole."

$$g(2) = \frac{2(2) + 3}{2} = \frac{7}{2}$$

Hence, there will be a "hole" in the graph of f at $(2, 7/2)$.

Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a) (b) (c)

19. **Step 1:** Factor.

$$f(x) = \frac{4 + 2x - 2x^2}{x^2 + 4x + 3} = \frac{-2(x^2 - x - 2)}{x^2 + 4x + 3} = \frac{-2(x - 2)(x + 1)}{(x + 3)(x + 1)}$$

Step 2: The restrictions are $x = -3$ and $x = -1$.

Step 3: The number $x = 2$ makes the numerator zero without making the denominator zero. Hence, $x = 2$ is a zero of f and $(2, 0)$ is an x-intercept of the graph of f.

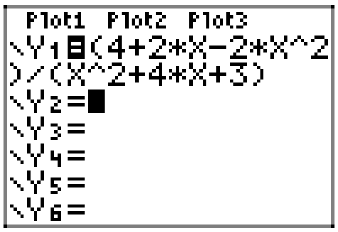
Step 4: Reduce, obtaining the new function

$$g(x) = \frac{-2(x - 2)}{x + 3}.$$

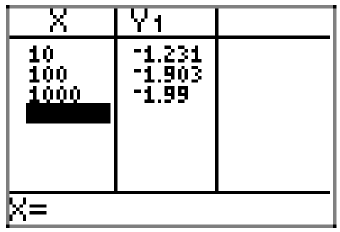
Note that $x = -3$ is still a restriction of the reduced form, so the graph of f must have vertical asymptote with equation $x = -3$. Note that $x = -1$ is no longer a restriction of the reduced form, so the graph of f will have a "hole" at this restriction. We will deal with the coordinates of this point in a moment.

Step 5: We will calculate and plot points, one on each side of the vertical asymptote: $(-4, -12)$ and $(-2, 8)$.

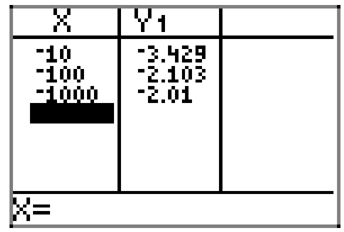
Step 6: To find the horizontal asymptote, we use our calculator (see images (a), (b), and (c) below) to determine the end-behavior of f.



(a)



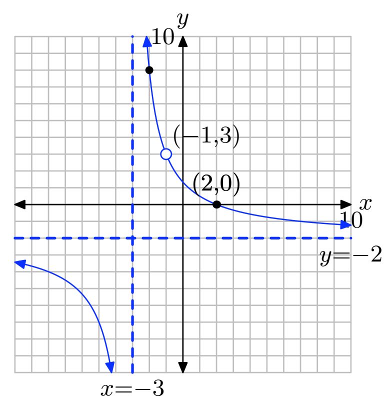
(b)



(c)

Thus, the line $y = -2$ is a horizontal asymptote.

Step 7: Putting all of this information together allows us to draw the following graph.

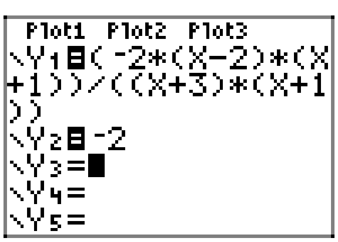


Step 8: Recall that the graph of f will have a "hole" at the restriction $x = -1$. Use the reduced form $g(x) = (-2(x-2))/(x+3)$ to compute the y-value of the hole.

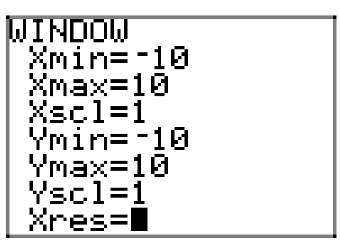
$$g(-1) = \frac{-2(-1-2)}{-1+3} = 3$$

Hence, there will be a "hole" in the graph of f at $(-1, 3)$.

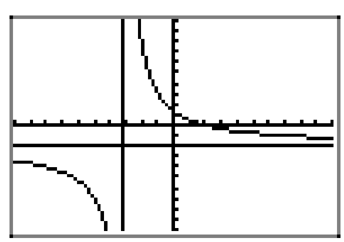
Step 9: We can use our graphing calculator to check our graph, as shown in the following sequence of images.



(a)



(b)



(c)

21. x-intercepts occur at values where the numerator of the function is zero, but the denominator is not zero.

$$\frac{81-x^2}{x^2+10x+9} = -\frac{x^2-81}{x^2+10x+9} = -\frac{(x+9)(x-9)}{(x+9)(x+1)}$$

The numerator is zero at $x=-9$ and $x=9$, and the denominator is zero at $x=-9$ and $x=-1$. Therefore, the only x-intercept occurs at $x=9$.

23. x-intercepts occur at values where the numerator of the function is zero, but the denominator is not zero.

$$\frac{x^2-x-12}{x^2+2x-3} = \frac{(x+3)(x-4)}{(x+3)(x-1)}$$

The numerator is zero at $x=-3$ and $x=4$, and the denominator is zero at $x=-3$ and $x=1$. Therefore, the only x-intercept occurs at $x=4$.

25. x-intercepts occur at values where the numerator of the function is zero, but the denominator is not zero.

$$\frac{6x-18}{x^2-7x+12} = \frac{6(x-3)}{(x-3)(x-4)}$$

The numerator is zero at $x=3$, and the denominator is zero at $x=3$ and $x=4$. Therefore, there are no x-intercepts.

27. x-intercepts occur at values where the numerator of the function is zero, but the denominator is not zero.

$$\frac{x^2-9x+14}{x^2-2x} = \frac{(x-2)(x-7)}{x(x-2)}$$

The numerator is zero at $x=2$ and $x=7$, and the denominator is zero at $x=2$ and $x=0$. Therefore, the only x-intercept occurs at $x=7$.

29. Vertical asymptotes occur where the *simplified* function is not defined.

$$\frac{x^2-7x}{x^2-2x} = \frac{x(x-7)}{x(x-2)} = \frac{x-7}{x-2}$$

so the line $x=2$ is the only vertical asymptote.

31. Vertical asymptotes occur where the *simplified* function is not defined.

$$\frac{x^2-6x+8}{x^2-16} = \frac{(x-4)(x-2)}{(x-4)(x+4)} = \frac{x-2}{x+4}$$

so the line $x=-4$ is the only vertical asymptote.

33. Vertical asymptotes occur where the *simplified* function is not defined.

$$\frac{x^2+x-12}{-4x+12} = \frac{(x-3)(x+4)}{-4(x-3)} = -\frac{x+4}{4}$$

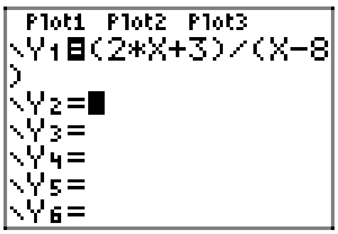
so there are no vertical asymptotes.

35. Vertical asymptotes occur where the *simplified* function is not defined.

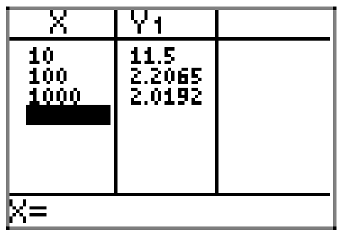
$$\frac{16-x^2}{x^2+7x+12} = -\frac{x^2-16}{x^2+7x+12} = -\frac{(x+4)(x-4)}{(x+4)(x+3)} = -\frac{x-4}{x+3}$$

so the line $x = -3$ is the only vertical asymptote.

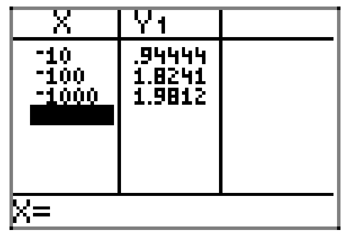
37. Load the equation in (a), then determine the end-behavior in (b) and (c).



(a)



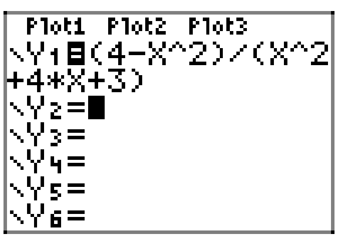
(b)



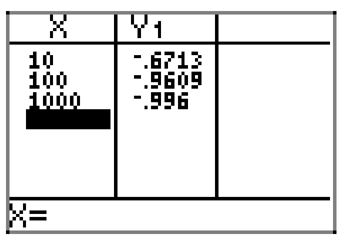
(c)

Hence, the equation of the horizontal asymptote is $y = 2$.

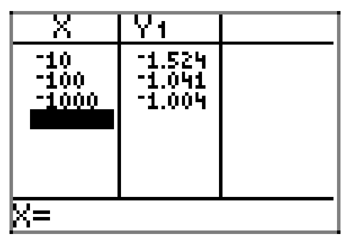
39. Load the equation in (a), then determine the end-behavior in (b) and (c).



(a)



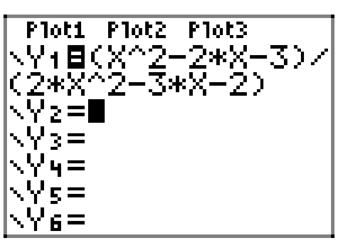
(b)



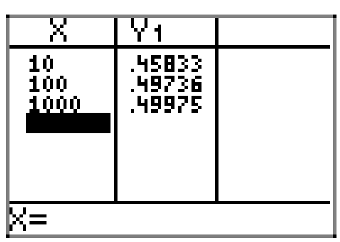
(c)

Hence, the equation of the horizontal asymptote is $y = -1$.

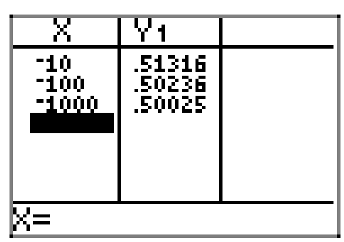
41. Load the equation in (a), then determine the end-behavior in (b) and (c).



(a)



(b)



(c)

Hence, the equation of the horizontal asymptote is $y = 1/2$.

43. The denominator factors as $-9(x+1)$, so an input of $x = -1$ would cause division by zero. Therefore, -1 is not in the domain. All other possible inputs are valid.

45. The denominator factors as $x(x-3)$, so an input of $x = 3$ or $x = 0$ would cause division by zero. Therefore, 3 and 0 are not in the domain. All other possible inputs are valid.

47. The denominator factors as $x(1-x)$, so inputs of $x = 0$ or $x = 1$ would cause division by zero. Therefore, 0 and 1 are not in the domain. All other possible inputs are valid.

7.4 Exercises

In **Exercises 1-10**, reduce the product to a single fraction in lowest terms.

1. $\dfrac{108}{14} \cdot \dfrac{6}{100}$

2. $\dfrac{75}{63} \cdot \dfrac{18}{45}$

3. $\dfrac{189}{56} \cdot \dfrac{12}{27}$

4. $\dfrac{45}{72} \cdot \dfrac{63}{64}$

5. $\dfrac{15}{36} \cdot \dfrac{28}{100}$

6. $\dfrac{189}{49} \cdot \dfrac{32}{25}$

7. $\dfrac{21}{100} \cdot \dfrac{125}{16}$

8. $\dfrac{21}{35} \cdot \dfrac{49}{45}$

9. $\dfrac{56}{20} \cdot \dfrac{98}{32}$

10. $\dfrac{27}{125} \cdot \dfrac{4}{12}$

In **Exercises 11-34**, multiply and simplify. State all restrictions.

11.

$$\frac{x+6}{x^2+16x+63} \cdot \frac{x^2+7x}{x+4}$$

12.

$$\frac{x^2+9x}{x^2-25} \cdot \frac{x^2-x-20}{-18-11x-x^2}$$

13.

$$\frac{x^2+7x+10}{x^2-1} \cdot \frac{-9+10x-x^2}{x^2+9x+20}$$

14.

$$\frac{x^2+5x}{x-4} \cdot \frac{x-2}{x^2+6x+5}$$

15.

$$\frac{x^2-5x}{x^2+2x-48} \cdot \frac{x^2+11x+24}{x^2-x}$$

16.

$$\frac{x^2-6x-27}{x^2+10x+24} \cdot \frac{x^2+13x+42}{x^2-11x+18}$$

17.

$$\frac{-x-x^2}{x^2-9x+8} \cdot \frac{x^2-4x+3}{x^2+4x+3}$$

18.

$$\frac{x^2-12x+35}{x^2+2x-15} \cdot \frac{45+4x-x^2}{x^2+x-30}$$

19.

$$\frac{x+2}{7-x} \cdot \frac{x^2+x-56}{x^2+7x+6}$$

20.

$$\frac{x^2-2x-15}{x^2+x} \cdot \frac{x^2+7x}{x^2+12x+27}$$

21.

$$\frac{x^2-9}{x^2-4x-45} \cdot \frac{x-6}{-3-x}$$

22.

$$\frac{x^2-12x+27}{x-4}\cdot\frac{x-5}{x^2-18x+81}$$

23.

$$\frac{x+5}{x^2+12x+32}\cdot\frac{x^2-2x-24}{x+7}$$

24.

$$\frac{x^2-36}{x^2+11x+24}\cdot\frac{-8-x}{x+4}$$

25.

$$\frac{x-5}{x^2-8x+12}\cdot\frac{x^2-12x+36}{x-8}$$

26.

$$\frac{x^2-5x-36}{x-1}\cdot\frac{x-5}{x^2-81}$$

27.

$$\frac{x^2+2x-15}{x^2-10x+16}\cdot\frac{x^2-7x+10}{3x^2+13x-10}$$

28.

$$\frac{5x^2+14x-3}{x+9}\cdot\frac{x-7}{x^2+10x+21}$$

29.

$$\frac{x^2-4}{x^2+2x-63}\cdot\frac{x^2+6x-27}{x^2-6x-16}$$

30.

$$\frac{x^2+5x+6}{x^2-3x}\cdot\frac{x^2-5x}{x^2+9x+18}$$

31.

$$\frac{x-1}{x^2+2x-63}\cdot\frac{x^2-81}{x+4}$$

32.

$$\frac{x^2+9x}{x^2+7x+12}\cdot\frac{27+6x-x^2}{x^2-5x}$$

33.

$$\frac{5-x}{x+3}\cdot\frac{x^2+3x-18}{2x^2-7x-15}$$

34.

$$\frac{4x^2+21x+5}{18-7x-x^2}\cdot\frac{x^2+11x+18}{x^2-25}$$

In **Exercises 35-58**, divide and simplify. State all restrictions.

35.

$$\frac{\dfrac{x^2-14x+48}{x^2+10x+16}}{\dfrac{-24+11x-x^2}{x^2-x-72}}$$

36.

$$\frac{x-1}{x^2-14x+48}\div\frac{x+5}{x^2-3x-18}$$

37.

$$\frac{x^2-1}{x^2-7x+12}\div\frac{x^2+6x+5}{-24+10x-x^2}$$

38.

$$\frac{x^2-13x+42}{x^2-2x-63}\div\frac{x^2-x-42}{x^2+8x+7}$$

39.

$$\frac{x^2-25}{x+1}\div\frac{5x^2+23x-10}{x-3}$$

40.
$$\frac{\dfrac{x^2-3x}{x^2-7x+6}}{\dfrac{x^2-4x}{3x^2-11x-42}}$$

41.
$$\frac{\dfrac{x^2+10x+21}{x-4}}{\dfrac{x^2+3x}{x+8}}$$

42.
$$\frac{x^2+8x+15}{x^2-14x+45} \div \frac{x^2+11x+30}{-30+11x-x^2}$$

43.
$$\frac{\dfrac{x^2-6x-16}{x^2+x-42}}{\dfrac{x^2-64}{x^2+12x+35}}$$

44.
$$\frac{\dfrac{x^2+3x+2}{x^2-9x+18}}{\dfrac{x^2+7x+6}{x^2-6x}}$$

45.
$$\frac{\dfrac{x^2+12x+35}{x+4}}{\dfrac{x^2+10x+25}{x+9}}$$

46.
$$\frac{x^2-8x+7}{x^2+3x-18} \div \frac{x^2-7x}{x^2+6x-27}$$

47.
$$\frac{x^2+x-30}{x^2+5x-36} \div \frac{-6-x}{x+8}$$

48.
$$\frac{\dfrac{2x-x^2}{x^2-15x+54}}{\dfrac{x^2+x}{x^2-11x+30}}$$

49.
$$\frac{\dfrac{x^2-9x+8}{x^2-9}}{\dfrac{x^2-8x}{-15-8x-x^2}}$$

50.
$$\frac{x+5}{x^2+2x+1} \div \frac{x-2}{x^2+10x+9}$$

51.
$$\frac{\dfrac{x^2-4}{x+8}}{\dfrac{x^2-10x+16}{x+3}}$$

52.
$$\frac{27-6x-x^2}{x^2+9x+20} \div \frac{x^2-12x+27}{x^2+5x}$$

53.
$$\frac{\dfrac{x^2+5x+6}{x^2-36}}{\dfrac{x-7}{-6-x}}$$

54.
$$\frac{2-x}{x-5} \div \frac{x^2+3x-10}{x^2-14x+48}$$

55.

$$\frac{\dfrac{x+3}{x^2+4x-12}}{\dfrac{x-4}{x^2-36}}$$

56.

$$\frac{x+3}{x^2-x-2} \div \frac{x}{x^2-3x-4}$$

57.

$$\frac{x^2-11x+28}{x^2+5x+6} \div \frac{7x^2-30x+8}{x^2-x-6}$$

58.

$$\frac{\dfrac{x-7}{3-x}}{\dfrac{2x^2+3x-5}{x^2-12x+27}}$$

---

59. Let

$$f(x) = \frac{x^2-7x+10}{x^2+4x-21}$$

and

$$g(x) = \frac{5x-x^2}{x^2+15x+56}$$

Compute $f(x)/g(x)$ and simplify your answer.

60. Let

$$f(x) = \frac{x^2+15x+56}{x^2-x-20}$$

and

$$g(x) = \frac{-7-x}{x+1}$$

Compute $f(x)/g(x)$ and simplify your answer.

61. Let

$$f(x) = \frac{x^2+12x+35}{x^2+4x-32}$$

and

$$g(x) = \frac{x^2-2x-35}{x^2+8x}$$

Compute $f(x)/g(x)$ and simplify your answer.

62. Let

$$f(x) = \frac{x^2+4x+3}{x-1}$$

and

$$g(x) = \frac{x^2-4x-21}{x+5}$$

Compute $f(x)/g(x)$ and simplify your answer.

63. Let

$$f(x) = \frac{x^2+x-20}{x}$$

and

$$g(x) = \frac{x-1}{x^2-2x-35}$$

Compute $f(x)g(x)$ and simplify your answer.

64. Let

$$f(x) = \frac{x^2+10x+24}{x^2-13x+42}$$

and

$$g(x) = \frac{x^2-6x-7}{x^2+8x+12}$$

Compute $f(x)g(x)$ and simplify your answer.

65. Let

$$f(x) = \frac{x+5}{-6-x}$$

and

$$g(x) = \frac{x^2+8x+12}{x^2-49}$$

Compute $f(x)g(x)$ and simplify your answer.

66. Let

$$f(x) = \frac{8-7x-x^2}{x^2-8x-9}$$

and

$$g(x) = \frac{x^2-6x-7}{x^2-6x+5}$$

Compute $f(x)g(x)$ and simplify your answer.

7.4 Solutions

1.

$$\frac{108}{14}\cdot\frac{6}{100}=\frac{2\cdot 2\cdot 3\cdot 3\cdot 3}{2\cdot 7}\cdot\frac{2\cdot 3}{2\cdot 2\cdot 5\cdot 5}=\frac{3\cdot 3\cdot 3\cdot 3}{7\cdot 5\cdot 5}=\frac{81}{175}$$

3.

$$\frac{189}{56}\cdot\frac{12}{27}=\frac{3\cdot 3\cdot 3\cdot 7}{2\cdot 2\cdot 2\cdot 7}\cdot\frac{2\cdot 2\cdot 3}{3\cdot 3\cdot 3}=\frac{3}{2}$$

5.

$$\frac{15}{36}\cdot\frac{28}{100}=\frac{3\cdot 5}{2\cdot 2\cdot 3\cdot 3}\cdot\frac{2\cdot 2\cdot 7}{2\cdot 2\cdot 5\cdot 5}=\frac{7}{3\cdot 2\cdot 2\cdot 5}=\frac{7}{60}$$

7.

$$\frac{21}{100}\cdot\frac{125}{16}=\frac{3\cdot 7}{2\cdot 2\cdot 5\cdot 5}\cdot\frac{5\cdot 5\cdot 5}{2\cdot 2\cdot 2\cdot 2}=\frac{3\cdot 7\cdot 5}{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}=\frac{105}{64}$$

9.

$$\frac{56}{20}\cdot\frac{98}{32}=\frac{2\cdot 2\cdot 2\cdot 7}{2\cdot 2\cdot 5}\cdot\frac{2\cdot 7\cdot 7}{2\cdot 2\cdot 2\cdot 2\cdot 2}=\frac{7\cdot 7\cdot 7}{5\cdot 2\cdot 2\cdot 2}=\frac{343}{40}$$

11. First factor the numerators and the denominators:

$$\frac{x+6}{(x+9)(x+7)}\cdot\frac{x(x+7)}{x+4}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{x(x+6)}{(x+9)(x+4)}$$

Restricted values are -9, -7, and -4.

13. First multiply $-9+10x-x^2$ by -1 and also negate the fraction:

$$-\frac{x^2+7x+10}{x^2-1}\cdot\frac{x^2-10x+9}{x^2+9x+20}$$

Then factor the numerators and the denominators:

$$-\frac{(x+2)(x+5)}{(x-1)(x+1)}\cdot\frac{(x-1)(x-9)}{(x+4)(x+5)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x+2)(x-9)}{(x+1)(x+4)}$$

Restricted values are 1, −1, −4, and −5.

15. First factor the numerators and the denominators:

$$\frac{x(x-5)}{(x+8)(x-6)} \cdot \frac{(x+8)(x+3)}{x(x-1)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-5)(x+3)}{(x-6)(x-1)}$$

Restricted values are −8, 6, 1, and 0.

17. First multiply $-x - x^2$ by −1 and also negate the fraction:

$$-\frac{x^2+x}{x^2-9x+8} \cdot \frac{x^2-4x+3}{x^2+4x+3}$$

Then factor the numerators and the denominators:

$$-\frac{x(x+1)}{(x-1)(x-8)} \cdot \frac{(x-1)(x-3)}{(x+3)(x+1)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{x(x-3)}{(x-8)(x+3)}$$

Restricted values are 1, 8, −3, and −1.

19. First multiply $7 - x$ by −1 and also negate the fraction:

$$-\frac{x+2}{x-7} \cdot \frac{x^2+x-56}{x^2+7x+6}$$

Then factor the numerators and the denominators:

$$-\frac{x+2}{x-7} \cdot \frac{(x-7)(x+8)}{(x+1)(x+6)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x+2)(x+8)}{(x+1)(x+6)}$$

Restricted values are 7, −1, and −6.

21. First multiply $-3-x$ by -1 and also negate the fraction:

$$-\frac{x^2-9}{x^2-4x-45}\cdot\frac{x-6}{x+3}$$

Then factor the numerators and the denominators:

$$-\frac{(x+3)(x-3)}{(x+5)(x-9)}\cdot\frac{x-6}{x+3}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-3)(x-6)}{(x+5)(x-9)}$$

Restricted values are -3, -5, and 9.

23. First factor the numerators and the denominators:

$$\frac{x+5}{(x+8)(x+4)}\cdot\frac{(x-6)(x+4)}{x+7}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+5)(x-6)}{(x+8)(x+7)}$$

Restricted values are -8, -4, and -7.

25. First factor the numerators and the denominators:

$$\frac{x-5}{(x-2)(x-6)}\cdot\frac{(x-6)^2}{x-8}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-5)(x-6)}{(x-2)(x-8)}$$

Restricted values are 2, 6, and 8.

27. First factor the numerators and the denominators:

$$\frac{(x-3)(x+5)}{(x-2)(x-8)}\cdot\frac{(x-2)(x-5)}{(3x-2)(x+5)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-3)(x-5)}{(3x-2)(x-8)}$$

Restricted values are 2, 8, 2/3, and -5.

29. First factor the numerators and the denominators:

$$\frac{(x-2)(x+2)}{(x+9)(x-7)} \cdot \frac{(x+9)(x-3)}{(x-8)(x+2)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-2)(x-3)}{(x-7)(x-8)}$$

Restricted values are -9, 7, 8, and -2.

31. First factor the numerators and the denominators:

$$\frac{x-1}{(x-7)(x+9)} \cdot \frac{(x-9)(x+9)}{x+4}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-1)(x-9)}{(x-7)(x+4)}$$

Restricted values are 7, -9, and -4.

33. First multiply $5-x$ by -1 and also negate the fraction:

$$-\frac{x-5}{x+3} \cdot \frac{x^2+3x-18}{2x^2-7x-15}$$

Then factor the numerators and the denominators:

$$-\frac{x-5}{x+3} \cdot \frac{(x+6)(x-3)}{(2x+3)(x-5)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x+6)(x-3)}{(2x+3)(x+3)}$$

Restricted values are -3, $-3/2$, and 5.

35. First rewrite as a multiplication problem:

$$\frac{x^2-14x+48}{x^2+10x+16} \cdot \frac{x^2-x-72}{-24+11x-x^2}$$

Then multiply $-24+11x-x^2$ by -1 and also negate the fraction:

$$-\frac{x^2-14x+48}{x^2+10x+16} \cdot \frac{x^2-x-72}{x^2-11x+24}$$

Then factor the numerators and the denominators:

$$-\frac{(x-6)(x-8)}{(x+8)(x+2)} \cdot \frac{(x+8)(x-9)}{(x-3)(x-8)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-6)(x-9)}{(x+2)(x-3)}$$

Restricted values are −8, −2, 9, 3, and 8.

37. First rewrite as a multiplication problem:

$$\frac{x^2-1}{x^2-7x+12}\cdot\frac{-24+10x-x^2}{x^2+6x+5}$$

Then multiply $-24+10x-x^2$ by −1 and also negate the fraction:

$$-\frac{x^2-1}{x^2-7x+12}\cdot\frac{x^2-10x+24}{x^2+6x+5}$$

Then factor the numerators and the denominators:

$$-\frac{(x-1)(x+1)}{(x-4)(x-3)}\cdot\frac{(x-4)(x-6)}{(x+5)(x+1)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-1)(x-6)}{(x-3)(x+5)}$$

Restricted values are 4, 3, 6, −5, and −1.

39. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x-5)(x+5)}{x+1}\cdot\frac{x-3}{(5x-2)(x+5)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-5)(x-3)}{(5x-2)(x+1)}$$

Restricted values are −1, 2/5, −5, and 3.

41. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x+7)(x+3)}{x-4}\cdot\frac{x+8}{x(x+3)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+7)(x+8)}{x(x-4)}$$

Restricted values are 4, 0, −3, and −8.

43. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x+2)(x-8)}{(x+7)(x-6)} \cdot \frac{(x+7)(x+5)}{(x+8)(x-8)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+2)(x+5)}{(x-6)(x+8)}$$

Restricted values are -7, 6, -5, -8, and 8.

45. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x+7)(x+5)}{x+4} \cdot \frac{x+9}{(x+5)^2}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+7)(x+9)}{(x+4)(x+5)}$$

Restricted values are -4, -5, and -9.

47. First rewrite as a multiplication problem:

$$\frac{x^2+x-30}{x^2+5x-36} \cdot \frac{x+8}{-6-x}$$

Then multiply $-6-x$ by -1 and also negate the fraction:

$$-\frac{x^2+x-30}{x^2+5x-36} \cdot \frac{x+8}{x+6}$$

Then factor the numerators and the denominators:

$$-\frac{(x+6)(x-5)}{(x-4)(x+9)} \cdot \frac{x+8}{x+6}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-5)(x+8)}{(x-4)(x+9)}$$

Restricted values are 4, -9, -8, and -6.

49. First rewrite as a multiplication problem:

$$\frac{x^2-9x+8}{x^2-9}\cdot\frac{-15-8x-x^2}{x^2-8x}$$

Then multiply $-15-8x-x^2$ by -1 and also negate the fraction:

$$-\frac{x^2-9x+8}{x^2-9}\cdot\frac{x^2+8x+15}{x^2-8x}$$

Then factor the numerators and the denominators:

$$-\frac{(x-1)(x-8)}{(x+3)(x-3)}\cdot\frac{(x+3)(x+5)}{x(x-8)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-1)(x+5)}{x(x-3)}$$

Restricted values are -3, 3, -5, 0, and 8.

51. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x+2)(x-2)}{x+8}\cdot\frac{x+3}{(x-8)(x-2)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+2)(x+3)}{(x+8)(x-8)}$$

Restricted values are -8, 8, 2, and -3.

53. First rewrite as a multiplication problem:

$$\frac{x^2+5x+6}{x^2-36}\cdot\frac{-6-x}{x-7}$$

Then multiply $-6-x$ by -1 and also negate the fraction:

$$-\frac{x^2+5x+6}{x^2-36}\cdot\frac{x+6}{x-7}$$

Then factor the numerators and the denominators:

$$-\frac{(x+2)(x+3)}{(x-6)(x+6)}\cdot\frac{x+6}{x-7}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x+2)(x+3)}{(x-6)(x-7)}$$

Restricted values are 6, -6, and 7.

55. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{x+3}{(x-2)(x+6)} \cdot \frac{(x-6)(x+6)}{x-4}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x+3)(x-6)}{(x-2)(x-4)}$$

Restricted values are 2, −6, 4, and 6.

57. First factor the numerators and the denominators, and rewrite as a multiplication problem:

$$\frac{(x-7)(x-4)}{(x+2)(x+3)} \cdot \frac{(x+2)(x-3)}{(7x-2)(x-4)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-7)(x-3)}{(7x-2)(x+3)}$$

Restricted values are −2, −3, 3, 2/7, and 4.

59.

$$\frac{f(x)}{g(x)} = \frac{x^2-7x+10}{x^2+4x-21} \cdot \frac{x^2+15x+56}{5x-x^2}$$

First multiply $5x - x^2$ by −1 and also negate the fraction:

$$-\frac{x^2-7x+10}{x^2+4x-21} \cdot \frac{x^2+15x+56}{x^2-5x}$$

Then factor the numerators and the denominators:

$$-\frac{(x-2)(x-5)}{(x+7)(x-3)} \cdot \frac{(x+7)(x+8)}{x(x-5)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x-2)(x+8)}{x(x-3)}$$

Restricted values are −7, 3, −8, 0, and 5.

61.

$$\frac{f(x)}{g(x)} = \frac{x^2+12x+35}{x^2+4x-32} \cdot \frac{x^2+8x}{x^2-2x-35}$$

Factor the numerators and the denominators:

$$\frac{(x+7)(x+5)}{(x+8)(x-4)} \cdot \frac{x(x+8)}{(x-7)(x+5)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{x(x+7)}{(x-4)(x-7)}$$

Restricted values are -8, 4, 0, 7, and -5.

63.

$$f(x)g(x) = \frac{x^2+x-20}{x} \cdot \frac{x-1}{x^2-2x-35}$$

First factor the numerators and the denominators:

$$\frac{(x-4)(x+5)}{x} \cdot \frac{x-1}{(x-7)(x+5)}$$

Then cancel all common factors and multiply the two remaining fractions:

$$\frac{(x-4)(x-1)}{x(x-7)}$$

Restricted values are 0, 7, and -5.

65.

$$f(x)g(x) = \frac{x+5}{-6-x} \cdot \frac{x^2+8x+12}{x^2-49}$$

First multiply $-6-x$ by -1 and also negate the fraction:

$$-\frac{x+5}{x+6} \cdot \frac{x^2+8x+12}{x^2-49}$$

Then factor the numerators and the denominators:

$$-\frac{x+5}{x+6} \cdot \frac{(x+6)(x+2)}{(x+7)(x-7)}$$

Finally, cancel all common factors and multiply the two remaining fractions:

$$-\frac{(x+5)(x+2)}{(x+7)(x-7)}$$

Restricted values are -6, -7, and 7.

7.5 Exercises

In **Exercises 1-16**, add or subtract the rational expressions, as indicated, and simplify your answer. State all restrictions.

1. $\dfrac{7x^2-49x}{x-6}+\dfrac{42}{x-6}$

2. $\dfrac{2x^2-110}{x-7}-\dfrac{12}{7-x}$

3. $\dfrac{27x-9x^2}{x+3}+\dfrac{162}{x+3}$

4. $\dfrac{2x^2-28}{x+2}-\dfrac{10x}{x+2}$

5. $\dfrac{4x^2-8}{x-4}+\dfrac{56}{4-x}$

6. $\dfrac{4x^2}{x-2}-\dfrac{36x-56}{x-2}$

7. $\dfrac{9x^2}{x-1}+\dfrac{72x-63}{1-x}$

8. $\dfrac{5x^2+30}{x-6}-\dfrac{35x}{x-6}$

9. $\dfrac{4x^2-60x}{x-7}+\dfrac{224}{x-7}$

10. $\dfrac{3x^2}{x-7}-\dfrac{63-30x}{7-x}$

11. $\dfrac{3x^2}{x-2}-\dfrac{48-30x}{2-x}$

12. $\dfrac{4x^2-164}{x-6}-\dfrac{20}{6-x}$

13. $\dfrac{9x^2}{x-2}-\dfrac{81x-126}{x-2}$

14. $\dfrac{9x^2}{x-8}+\dfrac{144x-576}{8-x}$

15. $\dfrac{3x^2-12}{x-3}+\dfrac{15}{3-x}$

16. $\dfrac{7x^2}{x-9}-\dfrac{112x-441}{x-9}$

In **Exercises 17-34**, add or subtract the rational expressions, as indicated, and simplify your answer. State all restrictions.

17. $\dfrac{3x}{x^2-6x+5}+\dfrac{15}{x^2-14x+45}$

18. $\dfrac{7x}{x^2-4x}+\dfrac{28}{x^2-12x+32}$

19. $\dfrac{9x}{x^2+4x-12}-\dfrac{54}{x^2+20x+84}$

20. $\dfrac{9x}{x^2-25}-\dfrac{45}{x^2+20x+75}$

21. $\dfrac{5x}{x^2-21x+98}-\dfrac{35}{7x-x^2}$

22. $\dfrac{7x}{7x-x^2}+\dfrac{147}{x^2+7x-98}$

23. $\dfrac{-7x}{x^2-8x+15}-\dfrac{35}{x^2-12x+35}$

24. $\dfrac{-6x}{x^2+2x}+\dfrac{12}{x^2+6x+8}$

25. $\dfrac{-9x}{x^2-12x+32}-\dfrac{36}{x^2-4x}$

26. $\dfrac{5x}{x^2-12x+32}-\dfrac{20}{4x-x^2}$

27. $\dfrac{6x}{x^2-21x+98}-\dfrac{42}{7x-x^2}$

28. $\dfrac{-2x}{x^2-3x-10}+\dfrac{4}{x^2+11x+18}$

29. $\dfrac{-9x}{x^2-6x+8}-\dfrac{18}{x^2-2x}$

30. $\dfrac{6x}{5x-x^2}+\dfrac{90}{x^2+5x-50}$

31. $\dfrac{8x}{5x-x^2}+\dfrac{120}{x^2+5x-50}$

32. $\dfrac{-5x}{x^2+5x}+\dfrac{25}{x^2+15x+50}$

33. $\dfrac{-5x}{x^2+x-30}+\dfrac{30}{x^2+23x+102}$

34. $\dfrac{9x}{x^2+12x+32}-\dfrac{36}{x^2+4x}$

---

35. Let

$$f(x)=\frac{8x}{x^2+6x+8}$$

and

$$g(x)=\frac{16}{x^2+2x}$$

Compute $f(x)-g(x)$ and simplify your answer.

36. Let

$$f(x)=\frac{-7x}{x^2+8x+12}$$

and

$$g(x)=\frac{42}{x^2+16x+60}$$

Compute $f(x)+g(x)$ and simplify your answer.

37. Let

$$f(x)=\frac{11x}{x^2+12x+32}$$

and

$$g(x)=\frac{44}{-4x-x^2}$$

Compute $f(x)+g(x)$ and simplify your answer.

38. Let

$$f(x)=\frac{8x}{x^2-6x}$$

and

$$g(x)=\frac{48}{x^2-18x+72}$$

Compute $f(x)+g(x)$ and simplify your answer.

39. Let

$$f(x)=\frac{4x}{-x-x^2}$$

and

$$g(x)=\frac{4}{x^2+3x+2}$$

Compute $f(x)+g(x)$ and simplify your answer.

40. Let

$$f(x)=\frac{5x}{x^2-x-12}$$

and

$$g(x)=\frac{15}{x^2+13x+30}$$

Compute $f(x)-g(x)$ and simplify your answer.

7.5 Solutions

1. Provided $x \neq 6$,

$$\begin{aligned}\frac{7x^2-49x}{x-6}+\frac{42}{x-6} &= \frac{7x^2-49x+42}{x-6}\\ &= \frac{7(x^2-7x+6)}{x-6}\\ &= \frac{7(x-6)(x-1)}{x-6}\\ &= 7(x-1)\end{aligned}$$

3. Provided $x \neq -3$,

$$\begin{aligned}\frac{27x-9x^2}{x+3}+\frac{162}{x+3} &= \frac{-9x^2+27x+162}{x+3}\\ &= \frac{-9(x^2-3x-18)}{x+3}\\ &= \frac{-9(x+3)(x-6)}{x+3}\\ &= -9(x-6)\end{aligned}$$

5. Provided $x \neq 4$,

$$\begin{aligned}\frac{4x^2-8}{x-4}+\frac{56}{4-x} &= \frac{4x^2-8}{x-4}-\frac{56}{x-4}\\ &= \frac{4x^2-8-56}{x-4}\\ &= \frac{4x^2-64}{x-4}\\ &= \frac{4(x^2-16)}{x-4}\\ &= \frac{4(x-4)(x+4)}{x-4}\\ &= 4(x+4)\end{aligned}$$

7. Provided $x \neq 1$,

$$\begin{aligned}\frac{9x^2}{x-1}+\frac{72x-63}{1-x} &= \frac{9x^2}{x-1}-\frac{72x-63}{x-1}\\ &= \frac{9x^2-72x+63}{x-1}\\ &= \frac{9(x^2-8x+7)}{x-1}\\ &= \frac{9(x-1)(x-7)}{x-1}\\ &= 9(x-7)\end{aligned}$$

9. Provided $x \neq 7$,

$$\begin{aligned}\frac{4x^2-60x}{x-7}+\frac{224}{x-7} &= \frac{4x^2-60x+224}{x-7}\\ &= \frac{4(x^2-15x+56)}{x-7}\\ &= \frac{4(x-7)(x-8)}{x-7}\\ &= 4(x-8)\end{aligned}$$

11. Provided $x \neq 2$,

$$\begin{aligned}\frac{3x^2}{x-2}-\frac{48-30x}{2-x} &= \frac{3x^2}{x-2}+\frac{48-30x}{x-2}\\ &= \frac{3x^2-30x+48}{x-2}\\ &= \frac{3(x^2-10x+16)}{x-2}\\ &= \frac{3(x-2)(x-8)}{x-2}\\ &= 3(x-8)\end{aligned}$$

13. Provided $x \neq 2$,

$$\begin{aligned}\frac{9x^2}{x-2}-\frac{81x-126}{x-2} &= \frac{9x^2-81x+126}{x-2}\\ &= \frac{9(x^2-9x+14)}{x-2}\\ &= \frac{9(x-2)(x-7)}{x-2}\\ &= 9(x-7)\end{aligned}$$

15. Provided $x \neq 3$,

$$\begin{aligned}\frac{3x^2-12}{x-3}+\frac{15}{3-x} &= \frac{3x^2-12}{x-3}-\frac{15}{x-3}\\ &= \frac{3x^2-12-15}{x-3}\\ &= \frac{3x^2-27}{x-3}\\ &= \frac{3(x^2-9)}{x-3}\\ &= \frac{3(x-3)(x+3)}{x-3}\\ &= 3(x+3)\end{aligned}$$

17.

$$\begin{aligned}&\frac{3x}{x^2-6x+5}+\frac{15}{x^2-14x+45}\\ &= \frac{3x}{(x-5)(x-1)}+\frac{15}{(x-5)(x-9)}\\ &= \frac{3x(x-9)}{(x-5)(x-1)(x-9)}+\frac{15(x-1)}{(x-5)(x-1)(x-9)}\\ &= \frac{3x(x-9)+15(x-1)}{(x-5)(x-1)(x-9)}\\ &= \frac{3x^2-12x-15}{(x-5)(x-1)(x-9)}\\ &= \frac{3(x-5)(x+1)}{(x-5)(x-1)(x-9)}\\ &= \frac{3(x+1)}{(x-1)(x-9)}\end{aligned}$$

Restricted values are 5, 1, and 9.

19.

$$
\begin{aligned}
&\frac{9x}{x^2+4x-12}-\frac{54}{x^2+20x+84}\\
&=\frac{9x}{(x+6)(x-2)}-\frac{54}{(x+6)(x+14)}\\
&=\frac{9x(x+14)}{(x+6)(x-2)(x+14)}-\frac{54(x-2)}{(x+6)(x-2)(x+14)}\\
&=\frac{9x(x+14)-54(x-2)}{(x+6)(x-2)(x+14)}\\
&=\frac{9x^2+72x+108}{(x+6)(x-2)(x+14)}\\
&=\frac{9(x+6)(x+2)}{(x+6)(x-2)(x+14)}\\
&=\frac{9(x+2)}{(x-2)(x+14)}
\end{aligned}
$$

Restricted values are -6, 2, and -14.

21.

$$
\begin{aligned}
&\frac{5x}{x^2-21x+98}-\frac{35}{7x-x^2}\\
&=\frac{5x}{x^2-21x+98}+\frac{35}{x^2-7x}\\
&=\frac{5x}{(x-7)(x-14)}+\frac{35}{x(x-7)}\\
&=\frac{5x^2}{x(x-7)(x-14)}+\frac{35(x-14)}{x(x-7)(x-14)}\\
&=\frac{5x^2+35(x-14)}{x(x-7)(x-14)}\\
&=\frac{5x^2+35x-490}{x(x-7)(x-14)}\\
&=\frac{5(x-7)(x+14)}{x(x-7)(x-14)}\\
&=\frac{5(x+14)}{x(x-14)}
\end{aligned}
$$

Restricted values are 7, 14, and 0.

23.

$$
\begin{aligned}
&\frac{-7x}{x^2-8x+15}-\frac{35}{x^2-12x+35}\\
&=\frac{-7x}{(x-5)(x-3)}-\frac{35}{(x-5)(x-7)}\\
&=\frac{-7x(x-7)}{(x-5)(x-3)(x-7)}-\frac{35(x-3)}{(x-5)(x-3)(x-7)}\\
&=\frac{-7x(x-7)-35(x-3)}{(x-5)(x-3)(x-7)}\\
&=\frac{-7x^2+14x+105}{(x-5)(x-3)(x-7)}\\
&=\frac{-7(x-5)(x+3)}{(x-5)(x-3)(x-7)}\\
&=\frac{-7(x+3)}{(x-3)(x-7)}
\end{aligned}
$$

Restricted values are 5, 3, and 7.

25.

$$
\begin{aligned}
&\frac{-9x}{x^2-12x+32}-\frac{36}{x^2-4x}\\
&=\frac{-9x}{(x-4)(x-8)}-\frac{36}{x(x-4)}\\
&=\frac{-9x^2}{x(x-4)(x-8)}-\frac{36(x-8)}{x(x-4)(x-8)}\\
&=\frac{-9x^2-36(x-8)}{x(x-4)(x-8)}\\
&=\frac{-9x^2-36x+288}{x(x-4)(x-8)}\\
&=\frac{-9(x-4)(x+8)}{x(x-4)(x-8)}\\
&=\frac{-9(x+8)}{x(x-8)}
\end{aligned}
$$

Restricted values are 4, 8, and 0.

27.

$$
\begin{aligned}
&\frac{6x}{x^2-21x+98}-\frac{42}{7x-x^2}\\
&=\frac{6x}{x^2-21x+98}+\frac{42}{x^2-7x}\\
&=\frac{6x}{(x-7)(x-14)}+\frac{42}{x(x-7)}\\
&=\frac{6x^2}{x(x-7)(x-14)}+\frac{42(x-14)}{x(x-7)(x-14)}\\
&=\frac{6x^2+42(x-14)}{x(x-7)(x-14)}\\
&=\frac{6x^2+42x-588}{x(x-7)(x-14)}\\
&=\frac{6(x-7)(x+14)}{x(x-7)(x-14)}\\
&=\frac{6(x+14)}{x(x-14)}
\end{aligned}
$$

Restricted values are 7, 14, and 0.

29.

$$
\begin{aligned}
&\frac{-9x}{x^2-6x+8}-\frac{18}{x^2-2x}\\
&=\frac{-9x}{(x-2)(x-4)}-\frac{18}{x(x-2)}\\
&=\frac{-9x^2}{x(x-2)(x-4)}-\frac{18(x-4)}{x(x-2)(x-4)}\\
&=\frac{-9x^2-18(x-4)}{x(x-2)(x-4)}\\
&=\frac{-9x^2-18x+72}{x(x-2)(x-4)}\\
&=\frac{-9(x-2)(x+4)}{x(x-2)(x-4)}\\
&=\frac{-9(x+4)}{x(x-4)}
\end{aligned}
$$

Restricted values are 2, 4, and 0.

31.

$$\begin{aligned}
&\frac{8x}{5x-x^2}+\frac{120}{x^2+5x-50}\\
&=\frac{-8x}{x^2-5x}+\frac{120}{x^2+5x-50}\\
&=\frac{-8x}{x(x-5)}+\frac{120}{(x-5)(x+10)}\\
&=\frac{-8x(x+10)}{x(x-5)(x+10)}+\frac{120x}{x(x-5)(x+10)}\\
&=\frac{-8x(x+10)+120x}{x(x-5)(x+10)}\\
&=\frac{-8x^2+40x}{x(x-5)(x+10)}\\
&=\frac{-8x(x-5)}{x(x-5)(x+10)}\\
&=\frac{-8}{x+10}
\end{aligned}$$

Restricted values are 5, 0, and -10.

33.

$$\begin{aligned}
&\frac{-5x}{x^2+x-30}+\frac{30}{x^2+23x+102}\\
&=\frac{-5x}{(x+6)(x-5)}+\frac{30}{(x+6)(x+17)}\\
&=\frac{-5x(x+17)}{(x+6)(x-5)(x+17)}+\frac{30(x-5)}{(x+6)(x-5)(x+17)}\\
&=\frac{-5x(x+17)+30(x-5)}{(x+6)(x-5)(x+17)}\\
&=\frac{-5x^2-55x-150}{(x+6)(x-5)(x+17)}\\
&=\frac{-5(x+6)(x+5)}{(x+6)(x-5)(x+17)}\\
&=\frac{-5(x+5)}{(x-5)(x+17)}
\end{aligned}$$

Restricted values are -6, 5, and -17.

35.

$$\begin{aligned}
f(x) - g(x) &= \frac{8x}{x^2+6x+8} - \frac{16}{x^2+2x} \\
&= \frac{8x}{(x+2)(x+4)} - \frac{16}{x(x+2)} \\
&= \frac{8x^2}{x(x+2)(x+4)} - \frac{16(x+4)}{x(x+2)(x+4)} \\
&= \frac{8x^2 - 16(x+4)}{x(x+2)(x+4)} \\
&= \frac{8x^2 - 16x - 64}{x(x+2)(x+4)} \\
&= \frac{8(x+2)(x-4)}{x(x+2)(x+4)} \\
&= \frac{8(x-4)}{x(x+4)}
\end{aligned}$$

Restricted values are -2, -4, and 0.

37.

$$\begin{aligned}
f(x) + g(x) &= \frac{11x}{x^2+12x+32} + \frac{44}{-4x-x^2} \\
&= \frac{11x}{x^2+12x+32} - \frac{44}{x^2+4x} \\
&= \frac{11x}{(x+4)(x+8)} - \frac{44}{x(x+4)} \\
&= \frac{11x^2}{x(x+4)(x+8)} - \frac{44(x+8)}{x(x+4)(x+8)} \\
&= \frac{11x^2 - 44(x+8)}{x(x+4)(x+8)} \\
&= \frac{11x^2 - 44x - 352}{x(x+4)(x+8)} \\
&= \frac{11(x+4)(x-8)}{x(x+4)(x+8)} \\
&= \frac{11(x-8)}{x(x+8)}
\end{aligned}$$

Restricted values are -4, -8, and 0.

39.

$$\begin{aligned}
f(x)+g(x) &= \frac{4x}{-x-x^2}+\frac{4}{x^2+3x+2}\\
&= \frac{-4x}{x^2+x}+\frac{4}{x^2+3x+2}\\
&= \frac{-4x}{x(x+1)}+\frac{4}{(x+1)(x+2)}\\
&= \frac{-4x(x+2)}{x(x+1)(x+2)}+\frac{4x}{x(x+1)(x+2)}\\
&= \frac{-4x(x+2)+4x}{x(x+1)(x+2)}\\
&= \frac{-4x^2-4x}{x(x+1)(x+2)}\\
&= \frac{-4x(x+1)}{x(x+1)(x+2)}\\
&= \frac{-4x}{x(x+2)}\\
&= \frac{-4}{x+2}
\end{aligned}$$

Restricted values are -1, 0, and -2.

7.6 Exercises

In **Exercises 1-6**, evaluate the function at the given rational number. Then use the first or second technique for simplifying complex fractions explained in the narrative to simplify your answer.

1. Given

$$f(x)=\frac{x+1}{2-x},$$

evaluate and simplify $f(1/2)$.

2. Given

$$f(x)=\frac{2-x}{x+5},$$

evaluate and simplify $f(3/2)$.

3. Given

$$f(x)=\frac{2x+3}{4-x},$$

evaluate and simplify $f(1/3)$.

4. Given

$$f(x)=\frac{3-2x}{x+5},$$

evaluate and simplify $f(2/5)$.

5. Given

$$f(x)=\frac{5-2x}{x+4},$$

evaluate and simplify $f(3/5)$.

6. Given

$$f(x)=\frac{2x-9}{11-x},$$

evaluate and simplify $f(4/3)$.

In **Exercises 7-46**, simplify the given complex rational expression. State all restrictions.

7.

$$\frac{5+\dfrac{6}{x}}{\dfrac{25}{x}-\dfrac{36}{x^3}}$$

8.

$$\frac{7+\dfrac{9}{x}}{\dfrac{49}{x}-\dfrac{81}{x^3}}$$

9.

$$\frac{\dfrac{7}{x-2}-\dfrac{5}{x-7}}{\dfrac{8}{x-7}+\dfrac{3}{x+8}}$$

10.

$$\frac{\dfrac{9}{x+4}-\dfrac{7}{x-9}}{\dfrac{9}{x-9}+\dfrac{5}{x-4}}$$

11.

$$\frac{3+\dfrac{7}{x}}{\dfrac{9}{x^2}-\dfrac{49}{x^4}}$$

[1] Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

12.

$$\frac{2-\dfrac{5}{x}}{\dfrac{4}{x^2}-\dfrac{25}{x^4}}$$

13.

$$\frac{\dfrac{9}{x+4}+\dfrac{7}{x+9}}{\dfrac{9}{x+9}+\dfrac{2}{x-8}}$$

14.

$$\frac{\dfrac{4}{x-6}+\dfrac{9}{x-9}}{\dfrac{9}{x-6}+\dfrac{8}{x-9}}$$

15.

$$\frac{\dfrac{5}{x-7}-\dfrac{4}{x-4}}{\dfrac{10}{x-4}-\dfrac{5}{x+2}}$$

16.

$$\frac{\dfrac{3}{x+6}+\dfrac{7}{x+9}}{\dfrac{9}{x+6}-\dfrac{4}{x+9}}$$

17.

$$\frac{\dfrac{6}{x-3}+\dfrac{5}{x-8}}{\dfrac{9}{x-3}+\dfrac{7}{x-8}}$$

18.

$$\frac{\dfrac{7}{x-7}-\dfrac{4}{x-2}}{\dfrac{7}{x-7}-\dfrac{6}{x-2}}$$

19.

$$\frac{\dfrac{4}{x-2}+\dfrac{7}{x-7}}{\dfrac{5}{x-2}+\dfrac{2}{x-7}}$$

20.

$$\frac{\dfrac{9}{x+2}-\dfrac{7}{x+5}}{\dfrac{4}{x+2}+\dfrac{3}{x+5}}$$

21.

$$\frac{5+\dfrac{4}{x}}{\dfrac{25}{x}-\dfrac{16}{x^3}}$$

22.

$$\frac{\dfrac{6}{x+5}+\dfrac{5}{x+4}}{\dfrac{8}{x+5}-\dfrac{3}{x+4}}$$

23.

$$\frac{\dfrac{9}{x-5}+\dfrac{8}{x+4}}{\dfrac{5}{x-5}-\dfrac{4}{x+4}}$$

24.

$$\frac{\dfrac{4}{x-6}+\dfrac{4}{x-9}}{\dfrac{6}{x-6}+\dfrac{6}{x-9}}$$

25.

$$\frac{\dfrac{6}{x+8}+\dfrac{5}{x-2}}{\dfrac{5}{x-2}-\dfrac{2}{x+2}}$$

26.

$$\frac{\dfrac{7}{x+9}+\dfrac{9}{x-2}}{\dfrac{4}{x-2}+\dfrac{7}{x+1}}$$

27.

$$\frac{\dfrac{7}{x+7}-\dfrac{5}{x+4}}{\dfrac{8}{x+7}-\dfrac{3}{x+4}}$$

28.

$$\frac{25-\dfrac{16}{x^2}}{5+\dfrac{4}{x}}$$

29.

$$\frac{\dfrac{64}{x}-\dfrac{25}{x^3}}{8-\dfrac{5}{x}}$$

30.

$$\frac{\dfrac{4}{x+2}+\dfrac{5}{x-6}}{\dfrac{7}{x-6}-\dfrac{5}{x+7}}$$

31.

$$\frac{\dfrac{2}{x-6}-\dfrac{4}{x+9}}{\dfrac{3}{x-6}-\dfrac{6}{x+9}}$$

32.

$$\frac{\dfrac{3}{x+6}-\dfrac{4}{x+4}}{\dfrac{6}{x+6}-\dfrac{8}{x+4}}$$

33.

$$\frac{\dfrac{9}{x^2}-\dfrac{64}{x^4}}{3-\dfrac{8}{x}}$$

34.

$$\frac{\dfrac{9}{x^2}-\dfrac{25}{x^4}}{3-\dfrac{5}{x}}$$

35.

$$\frac{\dfrac{4}{x-4}-\dfrac{8}{x-7}}{\dfrac{4}{x-7}+\dfrac{2}{x+2}}$$

36.

$$\frac{2-\dfrac{7}{x}}{4-\dfrac{49}{x^2}}$$

37.

$$\frac{\dfrac{3}{x^2+8x-9}+\dfrac{3}{x^2-81}}{\dfrac{9}{x^2-81}+\dfrac{9}{x^2-8x-9}}$$

38.

$$\frac{\dfrac{7}{x^2-5x-14}+\dfrac{2}{x^2-7x-18}}{\dfrac{5}{x^2-7x-18}+\dfrac{8}{x^2-6x-27}}$$

39.

$$\frac{\dfrac{2}{x^2+8x+7}+\dfrac{5}{x^2+13x+42}}{\dfrac{7}{x^2+13x+42}+\dfrac{6}{x^2+3x-18}}$$

40.
$$\frac{\dfrac{3}{x^2+5x-14}+\dfrac{3}{x^2-7x-98}}{\dfrac{3}{x^2-7x-98}+\dfrac{3}{x^2-15x+14}}$$

41.
$$\frac{\dfrac{6}{x^2+11x+24}-\dfrac{6}{x^2+13x+40}}{\dfrac{9}{x^2+13x+40}-\dfrac{9}{x^2-3x-40}}$$

42.
$$\frac{\dfrac{7}{x^2+13x+30}+\dfrac{7}{x^2+19x+90}}{\dfrac{9}{x^2+19x+90}+\dfrac{9}{x^2+7x-18}}$$

43.
$$\frac{\dfrac{7}{x^2-6x+5}+\dfrac{7}{x^2+2x-35}}{\dfrac{8}{x^2+2x-35}+\dfrac{8}{x^2+8x+7}}$$

44.
$$\frac{\dfrac{2}{x^2-4x-12}-\dfrac{2}{x^2-x-30}}{\dfrac{2}{x^2-x-30}-\dfrac{2}{x^2-4x-45}}$$

45.
$$\frac{\dfrac{4}{x^2+6x-7}-\dfrac{4}{x^2+2x-3}}{\dfrac{4}{x^2+2x-3}-\dfrac{4}{x^2+5x+6}}$$

46.
$$\frac{\dfrac{9}{x^2+3x-4}+\dfrac{8}{x^2-7x+6}}{\dfrac{4}{x^2-7x+6}+\dfrac{9}{x^2-10x+24}}$$

47. Given $f(x)=2/x$, simplify
$$\frac{f(x)-f(3)}{x-3}.$$
State all restrictions.

48. Given $f(x)=5/x$, simplify
$$\frac{f(x)-f(2)}{x-2}.$$
State all restrictions.

49. Given $f(x)=3/x^2$, simplify
$$\frac{f(x)-f(1)}{x-1}.$$
State all restrictions.

50. Given $f(x)=5/x^2$, simplify
$$\frac{f(x)-f(2)}{x-2}.$$
State all restrictions.

51. Given $f(x)=7/x$, simplify
$$\frac{f(x+h)-f(x)}{h}.$$
State all restrictions.

52. Given $f(x)=4/x$, simplify
$$\frac{f(x+h)-f(x)}{h}.$$
State all restrictions.

53. Given
$$f(x)=\frac{x+1}{3-x},$$
find and simplify $f(1/x)$. State all restrictions.

54. Given

$$f(x) = \frac{2-x}{3x+4},$$

find and simplify $f(2/x)$. State all restrictions.

55. Given

$$f(x) = \frac{x+1}{2-5x},$$

find and simplify $f(5/x)$. State all restrictions.

56. Given

$$f(x) = \frac{2x-3}{4+x},$$

find and simplify $f(1/x)$. State all restrictions.

57. Given

$$f(x) = \frac{x}{x+2},$$

find and simplify $f(f(x))$. State all restrictions.

58. Given

$$f(x) = \frac{2x}{x+5},$$

find and simplify $f(f(x))$. State all restrictions.

7.6 Solutions

1.

$$f(1/2)=\frac{\frac{1}{2}+1}{2-\frac{1}{2}}=\frac{\frac{1}{2}+1}{2-\frac{1}{2}}\cdot\frac{2}{2}=\frac{1+2}{4-1}=\frac{3}{3}=1$$

3.

$$f(1/3)=\frac{2\left(\frac{1}{3}\right)+3}{4-\frac{1}{3}}=\frac{2\left(\frac{1}{3}\right)+3}{4-\frac{1}{3}}\cdot\frac{3}{3}=\frac{2+9}{12-1}=\frac{11}{11}=1$$

5.

$$f(3/5)=\frac{5-2\left(\frac{3}{5}\right)}{\frac{3}{5}+4}=\frac{5-2\left(\frac{3}{5}\right)}{\frac{3}{5}+4}\cdot\frac{5}{5}=\frac{25-6}{3+20}=\frac{19}{23}$$

7. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is x^3:

$$\frac{\left(5+\dfrac{6}{x}\right)\cdot\dfrac{x^3}{1}}{\left(\dfrac{25}{x}-\dfrac{36}{x^3}\right)\cdot\dfrac{x^3}{1}}=\frac{5x^3+6x^2}{25x^2-36}$$

Then factor the numerator and denominator:

$$\frac{x^2(5x+6)}{(5x+6)(5x-6)}$$

Finally, cancel all common factors in the numerator and denominator:

$$\frac{x^2}{5x-6}$$

Restricted values are 0, $-6/5$, and $6/5$.

9. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-2)(x-7)(x+8)$:

$$\frac{\left(\dfrac{7}{x-2}-\dfrac{5}{x-7}\right)\cdot\dfrac{(x-2)(x-7)(x+8)}{1}}{\left(\dfrac{8}{x-7}+\dfrac{3}{x+8}\right)\cdot\dfrac{(x-2)(x-7)(x+8)}{1}}$$

$$=\frac{7(x-7)(x+8)-5(x-2)(x+8)}{8(x-2)(x+8)+3(x-2)(x-7)}$$

Then simplify the numerator and denominator:

$$\begin{aligned}\frac{7(x-7)(x+8)-5(x-2)(x+8)}{8(x-2)(x+8)+3(x-2)(x-7)} &= \frac{(7(x-7)-5(x-2))(x+8)}{(8(x+8)+3(x-7))(x-2)}\\ &= \frac{(2x-39)(x+8)}{(11x+43)(x-2)}\end{aligned}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are 2, 7, −8, and −43/11.

11. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is x^4:

$$\frac{\left(3+\dfrac{7}{x}\right)\cdot\dfrac{x^4}{1}}{\left(\dfrac{9}{x^2}-\dfrac{49}{x^4}\right)\cdot\dfrac{x^4}{1}} = \frac{3x^4+7x^3}{9x^2-49}$$

Then factor the numerator and denominator:

$$\frac{x^3(3x+7)}{(3x+7)(3x-7)}$$

Finally, cancel all common factors in the numerator and denominator:

$$\frac{x^3}{3x-7}$$

Restricted values are 0, −7/3, and 7/3.

13. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x+4)(x+9)(x-8)$:

$$\begin{aligned}&\frac{\left(\dfrac{9}{x+4}+\dfrac{7}{x+9}\right)\cdot\dfrac{(x+4)(x+9)(x-8)}{1}}{\left(\dfrac{9}{x+9}+\dfrac{2}{x-8}\right)\cdot\dfrac{(x+4)(x+9)(x-8)}{1}}\\ &= \frac{9(x+9)(x-8)+7(x+4)(x-8)}{9(x+4)(x-8)+2(x+4)(x+9)}\end{aligned}$$

Then simplify the numerator and denominator:

$$\begin{aligned}\frac{9(x+9)(x-8)+7(x+4)(x-8)}{9(x+4)(x-8)+2(x+4)(x+9)} &= \frac{(9(x+9)+7(x+4))(x-8)}{(9(x-8)+2(x+9))(x+4)}\\ &= \frac{(16x+109)(x-8)}{(11x-54)(x+4)}\end{aligned}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are −4, −9, 8, and 54/11.

15. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-7)(x-4)(x+2)$:

$$\frac{\left(\frac{5}{x-7}-\frac{4}{x-4}\right)\cdot\frac{(x-7)(x-4)(x+2)}{1}}{\left(\frac{10}{x-4}-\frac{5}{x+2}\right)\cdot\frac{(x-7)(x-4)(x+2)}{1}}$$

$$=\frac{5(x-4)(x+2)-4(x-7)(x+2)}{10(x-7)(x+2)-5(x-7)(x-4)}$$

Then simplify the numerator and denominator:

$$\frac{5(x-4)(x+2)-4(x-7)(x+2)}{10(x-7)(x+2)-5(x-7)(x-4)}=\frac{(5(x-4)-4(x-7))(x+2)}{(10(x+2)-5(x-4))(x-7)}$$

$$=\frac{(x+8)(x+2)}{(5x+40)(x-7)}$$

Finally, cancel all common factors, if any, in the numerator and denominator:

$$\frac{(x+8)(x+2)}{(5x+40)(x-7)}=\frac{x+2}{5(x-7)}$$

Restricted values are 7, 4, −2, and −8.

17. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-3)(x-8)$:

$$\frac{\left(\frac{6}{x-3}+\frac{5}{x-8}\right)\cdot\frac{(x-3)(x-8)}{1}}{\left(\frac{9}{x-3}+\frac{7}{x-8}\right)\cdot\frac{(x-3)(x-8)}{1}}=\frac{6(x-8)+5(x-3)}{9(x-8)+7(x-3)}$$

Then simplify the numerator and denominator:

$$\frac{6(x-8)+5(x-3)}{9(x-8)+7(x-3)}=\frac{11x-63}{16x-93}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are 3, 8, and 93/16.

19. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-2)(x-7)$:

$$\frac{\left(\dfrac{4}{x-2}+\dfrac{7}{x-7}\right)\cdot\dfrac{(x-2)(x-7)}{1}}{\left(\dfrac{5}{x-2}+\dfrac{2}{x-7}\right)\cdot\dfrac{(x-2)(x-7)}{1}}=\frac{4(x-7)+7(x-2)}{5(x-7)+2(x-2)}$$

Then simplify the numerator and denominator:

$$\frac{4(x-7)+7(x-2)}{5(x-7)+2(x-2)}=\frac{11x-42}{7x-39}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are 2, 7, and 39/7.

21. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is x^3:

$$\frac{\left(5+\dfrac{4}{x}\right)\cdot\dfrac{x^3}{1}}{\left(\dfrac{25}{x}-\dfrac{16}{x^3}\right)\cdot\dfrac{x^3}{1}}=\frac{5x^3+4x^2}{25x^2-16}$$

Then factor the numerator and denominator:

$$\frac{x^2(5x+4)}{(5x+4)(5x-4)}$$

Finally, cancel all common factors in the numerator and denominator:

$$\frac{x^2}{5x-4}$$

Restricted values are 0, $-4/5$, and 4/5.

23. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-5)(x+4)$:

$$\frac{\left(\dfrac{9}{x-5}+\dfrac{8}{x+4}\right)\cdot\dfrac{(x-5)(x+4)}{1}}{\left(\dfrac{5}{x-5}-\dfrac{4}{x+4}\right)\cdot\dfrac{(x-5)(x+4)}{1}}=\frac{9(x+4)+8(x-5)}{5(x+4)-4(x-5)}$$

Then simplify the numerator and denominator:

$$\frac{9(x+4)+8(x-5)}{5(x+4)-4(x-5)}=\frac{17x-4}{x+40}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are 5, −4, and −40.

25. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x+8)(x-2)(x+2)$:

$$\frac{\left(\frac{6}{x+8}+\frac{5}{x-2}\right)\cdot\frac{(x+8)(x-2)(x+2)}{1}}{\left(\frac{5}{x-2}-\frac{2}{x+2}\right)\cdot\frac{(x+8)(x-2)(x+2)}{1}}$$
$$=\frac{6(x-2)(x+2)+5(x+8)(x+2)}{5(x+8)(x+2)-2(x+8)(x-2)}$$

Then simplify the numerator and denominator:

$$\frac{6(x-2)(x+2)+5(x+8)(x+2)}{5(x+8)(x+2)-2(x+8)(x-2)}=\frac{(6(x-2)+5(x+8))(x+2)}{(5(x+2)-2(x-2))(x+8)}$$
$$=\frac{(11x+28)(x+2)}{(3x+14)(x+8)}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are −8, 2, −2, and −14/3.

27. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x+7)(x+4)$:

$$\frac{\left(\frac{7}{x+7}-\frac{5}{x+4}\right)\cdot\frac{(x+7)(x+4)}{1}}{\left(\frac{8}{x+7}-\frac{3}{x+4}\right)\cdot\frac{(x+7)(x+4)}{1}}=\frac{7(x+4)-5(x+7)}{8(x+4)-3(x+7)}$$

Then simplify the numerator and denominator:

$$\frac{7(x+4)-5(x+7)}{8(x+4)-3(x+7)}=\frac{2x-7}{5x+11}$$

Finally, cancel all common factors, if any, in the numerator and denominator. In this case, there are no common factors.

Restricted values are −7, −4, and −11/5.

29. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is x^3:

$$\frac{\left(\frac{64}{x} - \frac{25}{x^3}\right) \cdot \frac{x^3}{1}}{\left(8 - \frac{5}{x}\right) \cdot \frac{x^3}{1}} = \frac{64x^2 - 25}{8x^3 - 5x^2}$$

Then factor the numerator and denominator:

$$\frac{(8x-5)(8x+5)}{x^2(8x-5)}$$

Finally, cancel all common factors in the numerator and denominator:

$$\frac{8x+5}{x^2}$$

Restricted values are 0 and 5/8.

31. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-6)(x+9)$:

$$\frac{\left(\frac{2}{x-6} - \frac{4}{x+9}\right) \cdot \frac{(x-6)(x+9)}{1}}{\left(\frac{3}{x-6} - \frac{6}{x+9}\right) \cdot \frac{(x-6)(x+9)}{1}} = \frac{2(x+9) - 4(x-6)}{3(x+9) - 6(x-6)}$$

Then simplify the numerator and denominator:

$$\frac{2(x+9) - 4(x-6)}{3(x+9) - 6(x-6)} = \frac{-2x+42}{-3x+63}$$

Finally, cancel all common factors, if any, in the numerator and denominator:

$$\frac{-2x+42}{-3x+63} = \frac{2}{3}$$

Restricted values are 6, −9, and 21.

33. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is x^4:

$$\frac{\left(\frac{9}{x^2} - \frac{64}{x^4}\right) \cdot \frac{x^4}{1}}{\left(3 - \frac{8}{x}\right) \cdot \frac{x^4}{1}} = \frac{9x^2 - 64}{3x^4 - 8x^3}$$

Then factor the numerator and denominator:

$$\frac{(3x-8)(3x+8)}{x^3(3x-8)}$$

Finally, cancel all common factors in the numerator and denominator:

$$\frac{3x+8}{x^3}$$

Restricted values are 0 and 8/3.

35. First multiply the main numerator and denominator by the least common denominator (LCD) of the two small fractions, which is $(x-4)(x-7)(x+2)$:

$$\frac{\left(\frac{4}{x-4} - \frac{8}{x-7}\right) \cdot \frac{(x-4)(x-7)(x+2)}{1}}{\left(\frac{4}{x-7} + \frac{2}{x+2}\right) \cdot \frac{(x-4)(x-7)(x+2)}{1}}$$
$$= \frac{4(x-7)(x+2) - 8(x-4)(x+2)}{4(x-4)(x+2) + 2(x-4)(x-7)}$$

Then simplify the numerator and denominator:

$$\begin{aligned}\frac{4(x-7)(x+2) - 8(x-4)(x+2)}{4(x-4)(x+2) + 2(x-4)(x-7)} &= \frac{(4(x-7) - 8(x-4))(x+2)}{(4(x+2) + 2(x-7))(x-4)} \\ &= \frac{(-4x+4)(x+2)}{(6x-6)(x-4)}\end{aligned}$$

Finally, cancel all common factors, if any, in the numerator and denominator:

$$\frac{(-4x+4)(x+2)}{(6x-6)(x-4)} = \frac{-2(x+2)}{3(x-4)}$$

Restricted values are 4, 7, −2, and 1.

37. First factor all of the denominators:

$$\frac{\dfrac{3}{(x-1)(x+9)}+\dfrac{3}{(x+9)(x-9)}}{\dfrac{9}{(x+9)(x-9)}+\dfrac{9}{(x-9)(x+1)}}$$

Then simplify the main numerator and denominator:

$$\frac{\dfrac{3(x-9)}{(x-1)(x+9)(x-9)}+\dfrac{3(x-1)}{(x-1)(x+9)(x-9)}}{\dfrac{9(x+1)}{(x+9)(x-9)(x+1)}+\dfrac{9(x+9)}{(x+9)(x-9)(x+1)}}$$

$$=\frac{\dfrac{3(x-9)+3(x-1)}{(x-1)(x+9)(x-9)}}{\dfrac{9(x+1)+9(x+9)}{(x+9)(x-9)(x+1)}}$$

$$=\frac{\dfrac{6x-30}{(x-1)(x+9)(x-9)}}{\dfrac{18x+90}{(x+9)(x-9)(x+1)}}$$

Then divide by rewriting as a multiplication problem, and cancel common factors:

$$\frac{6x-30}{(x-1)(x+9)(x-9)}\cdot\frac{(x+9)(x-9)(x+1)}{18x+90}$$

$$=\frac{(6x-30)(x+1)}{(18x+90)(x-1)}$$

Finally, factor the numerator and denominator further, if possible, and cancel all common factors again:

$$\frac{6(x-5)(x+1)}{18(x+5)(x-1)}$$

$$=\frac{(x-5)(x+1)}{3(x+5)(x-1)}$$

Restricted values are 1, -9, 9, -1, and -5.

39. First factor all of the denominators:

$$\frac{\dfrac{2}{(x+1)(x+7)}+\dfrac{5}{(x+7)(x+6)}}{\dfrac{7}{(x+7)(x+6)}+\dfrac{6}{(x+6)(x-3)}}$$

Then simplify the main numerator and denominator:

$$\frac{\dfrac{2(x+6)}{(x+1)(x+7)(x+6)}+\dfrac{5(x+1)}{(x+1)(x+7)(x+6)}}{\dfrac{7(x-3)}{(x+7)(x+6)(x-3)}+\dfrac{6(x+7)}{(x+7)(x+6)(x-3)}}$$

$$=\frac{\dfrac{2(x+6)+5(x+1)}{(x+1)(x+7)(x+6)}}{\dfrac{7(x-3)+6(x+7)}{(x+7)(x+6)(x-3)}}$$

$$=\frac{\dfrac{7x+17}{(x+1)(x+7)(x+6)}}{\dfrac{13x+21}{(x+7)(x+6)(x-3)}}$$

Then divide by rewriting as a multiplication problem, and cancel common factors:

$$\frac{7x+17}{(x+1)(x+7)(x+6)}\cdot\frac{(x+7)(x+6)(x-3)}{13x+21}$$

$$=\frac{(7x+17)(x-3)}{(13x+21)(x+1)}$$

Finally, factor the numerator and denominator further, if possible, and cancel all common factors again, if any. In this case, the expression does not simplify any further.

Restricted values are -1, -7, -6, 3, and $-21/13$.

41. First factor all of the denominators:

$$\frac{\dfrac{6}{(x+3)(x+8)} - \dfrac{6}{(x+8)(x+5)}}{\dfrac{9}{(x+8)(x+5)} - \dfrac{9}{(x+5)(x-8)}}$$

Then simplify the main numerator and denominator:

$$\begin{aligned}
&\frac{\dfrac{6(x+5)}{(x+3)(x+8)(x+5)} - \dfrac{6(x+3)}{(x+3)(x+8)(x+5)}}{\dfrac{9(x-8)}{(x+8)(x+5)(x-8)} - \dfrac{9(x+8)}{(x+8)(x+5)(x-8)}} \\
&= \frac{\dfrac{6(x+5)-6(x+3)}{(x+3)(x+8)(x+5)}}{\dfrac{9(x-8)-9(x+8)}{(x+8)(x+5)(x-8)}} \\
&= \frac{\dfrac{12}{(x+3)(x+8)(x+5)}}{\dfrac{-144}{(x+8)(x+5)(x-8)}}
\end{aligned}$$

Then divide by rewriting as a multiplication problem, and cancel common factors:

$$\begin{aligned}
&\frac{12}{(x+3)(x+8)(x+5)} \cdot \frac{(x+8)(x+5)(x-8)}{-144} \\
&= \frac{-1(x-8)}{12(x+3)}
\end{aligned}$$

Restricted values are -3, -8, -5, and 8.

43. First factor all of the denominators:

$$\frac{\dfrac{7}{(x-1)(x-5)}+\dfrac{7}{(x-5)(x+7)}}{\dfrac{8}{(x-5)(x+7)}+\dfrac{8}{(x+7)(x+1)}}$$

Then simplify the main numerator and denominator:

$$\begin{aligned}&\frac{\dfrac{7(x+7)}{(x-1)(x-5)(x+7)}+\dfrac{7(x-1)}{(x-1)(x-5)(x+7)}}{\dfrac{8(x+1)}{(x-5)(x+7)(x+1)}+\dfrac{8(x-5)}{(x-5)(x+7)(x+1)}}\\ &=\frac{\dfrac{7(x+7)+7(x-1)}{(x-1)(x-5)(x+7)}}{\dfrac{8(x+1)+8(x-5)}{(x-5)(x+7)(x+1)}}\\ &=\frac{\dfrac{14x+42}{(x-1)(x-5)(x+7)}}{\dfrac{16x-32}{(x-5)(x+7)(x+1)}}\end{aligned}$$

Then divide by rewriting as a multiplication problem, and cancel common factors:

$$\begin{aligned}&\frac{14x+42}{(x-1)(x-5)(x+7)}\cdot\frac{(x-5)(x+7)(x+1)}{16x-32}\\ &=\frac{(14x+42)(x+1)}{(16x-32)(x-1)}\end{aligned}$$

Finally, factor the numerator and denominator further, if possible, and cancel all common factors again:

$$\begin{aligned}&\frac{14(x+3)(x+1)}{16(x-2)(x-1)}\\ &=\frac{7(x+3)(x+1)}{8(x-2)(x-1)}\end{aligned}$$

Restricted values are 1, 5, −7, −1, and 2.

45. First factor all of the denominators:

$$\frac{\dfrac{4}{(x+7)(x-1)} - \dfrac{4}{(x-1)(x+3)}}{\dfrac{4}{(x-1)(x+3)} - \dfrac{4}{(x+3)(x+2)}}$$

Then simplify the main numerator and denominator:

$$\frac{\dfrac{4(x+3)}{(x+7)(x-1)(x+3)} - \dfrac{4(x+7)}{(x+7)(x-1)(x+3)}}{\dfrac{4(x+2)}{(x-1)(x+3)(x+2)} - \dfrac{4(x-1)}{(x-1)(x+3)(x+2)}}$$

$$= \frac{\dfrac{4(x+3)-4(x+7)}{(x+7)(x-1)(x+3)}}{\dfrac{4(x+2)-4(x-1)}{(x-1)(x+3)(x+2)}}$$

$$= \frac{\dfrac{-16}{(x+7)(x-1)(x+3)}}{\dfrac{12}{(x-1)(x+3)(x+2)}}$$

Then divide by rewriting as a multiplication problem, and cancel common factors:

$$\frac{-16}{(x+7)(x-1)(x+3)} \cdot \frac{(x-1)(x+3)(x+2)}{12}$$

$$= \frac{-4(x+2)}{3(x+7)}$$

Restricted values are -7, 1, -3, and -2.

47.

$$\frac{f(x)-f(3)}{x-3} = \frac{\frac{2}{x}-\frac{2}{3}}{x-3} = \frac{\frac{2}{x}-\frac{2}{3}}{x-3} \cdot \frac{3x}{3x} = \frac{6-2x}{3x(x-3)} = \frac{-2(x-3)}{3x(x-3)} = -\frac{2}{3x}$$

Restricted values are 0 and 3.

49.

$$\frac{f(x)-f(1)}{x-1} = \frac{\frac{3}{x^2}-3}{x-1} = \frac{\frac{3}{x^2}-3}{x-1} \cdot \frac{x^2}{x^2} = \frac{3-3x^2}{x^2(x-1)} = \frac{-3(x-1)(x+1)}{x^2(x-1)} = -\frac{3(x+1)}{x^2}$$

Restricted values are 0 and 1.

51.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{7}{x+h}-\frac{7}{x}}{h}=\frac{\frac{7}{x+h}-\frac{7}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}=\frac{7x-7(x+h)}{hx(x+h)}=\frac{-7h}{hx(x+h)}=-\frac{7}{h(x+h)}$$

Restricted values are $x \neq 0$, $x \neq -h$, and $h \neq 0$.

53.

$$f(1/x)=\frac{\frac{1}{x}+1}{3-\frac{1}{x}}=\frac{\frac{1}{x}+1}{3-\frac{1}{x}}\cdot\frac{x}{x}=\frac{1+x}{3x-1}=\frac{x+1}{3x-1}$$

Restricted values are 0 and 1/3.

55.

$$f(5/x)=\frac{\frac{5}{x}+1}{2-5\left(\frac{5}{x}\right)}=\frac{\frac{5}{x}+1}{2-5\left(\frac{5}{x}\right)}\cdot\frac{x}{x}=\frac{5+x}{2x-25}=\frac{x+5}{2x-25}$$

Restricted values are 0 and 25/2.

57.

$$f(f(x))=f\left(\frac{x}{x+2}\right)=\frac{\frac{x}{x+2}}{\frac{x}{x+2}+2}=\frac{\frac{x}{x+2}}{\frac{x}{x+2}+2}\cdot\frac{x+2}{x+2}=\frac{x}{x+2(x+2)}=\frac{x}{3x+4}$$

Restricted values are -2 and $-4/3$.

7.7 Exercises

For each of the rational functions given in **Exercises 1-6**, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis. *Remember to draw all lines with a ruler.*
ii. Plot the zero of the rational function on your coordinate system and label it with its coordinates. Plot the vertical and horizontal asymptotes on your coordinate system and label them with their equations. Use this information (and your graphing calculator) to draw the graph of f.
iii. Plot the horizontal line $y = k$ on your coordinate system and label this line with its equation.
iv. Use your calculator's `intersect` utility to help determine the solution of $f(x) = k$. Label this point on your graph with its coordinates.
v. Solve the equation $f(x) = k$ algebraically, placing the work for this solution on your graph paper next to your coordinate system containing the graphical solution. Do the answers agree?

1. $f(x) = \dfrac{x-1}{x+2}; \quad k = 3$

2. $f(x) = \dfrac{x+1}{x-2}; \quad k = -3$

3. $f(x) = \dfrac{x+1}{3-x}; \quad k = 2$

4. $f(x) = \dfrac{x+3}{2-x}; \quad k = 2$

5. $f(x) = \dfrac{2x+3}{x-1}; \quad k = -3$

6. $f(x) = \dfrac{5-2x}{x-1}; \quad k = 3$

In **Exercises 7-14**, use a strictly algebraic technique to solve the equation $f(x) = k$ for the given function and value of k. You are encouraged to check your result with your calculator.

7. $f(x) = \dfrac{16x-9}{2x-1}; \quad k = 8$

8. $f(x) = \dfrac{10x-3}{7x+7}; \quad k = 1$

9. $f(x) = \dfrac{5x+8}{4x+1}; \quad k = -11$

10. $f(x) = -\dfrac{6x-11}{7x-2}; \quad k = -6$

11. $f(x) = -\dfrac{35x}{7x+12}; \quad k = -5$

12. $f(x) = -\dfrac{66x-5}{6x-10}; \quad k = -11$

13. $f(x) = \dfrac{8x+2}{x-11}; \quad k = 11$

14. $f(x) = \dfrac{36x-7}{3x-4}; \quad k = 12$

In **Exercises 15-20**, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

15. $\dfrac{x}{7} + \dfrac{8}{9} = -\dfrac{8}{7}$

16. $\dfrac{x}{3} + \dfrac{9}{2} = -\dfrac{3}{8}$

17. $-\dfrac{57}{x} = 27 - \dfrac{40}{x^2}$

18. $-\dfrac{117}{x} = 54 + \dfrac{54}{x^2}$

19. $\dfrac{7}{x} = 4 - \dfrac{3}{x^2}$

20. $\dfrac{3}{x^2} = 5 - \dfrac{3}{x}$

For each of the rational functions given in **Exercises 21-26**, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis. *Remember to draw all lines with a ruler.*
ii. Plot the zero of the rational function on your coordinate system and label it with its coordinates. You may use your calculator's `zero` utility to find this, if you wish.
iii. Plot the vertical and horizontal asymptotes on your coordinate system and label them with their equations. Use the asymptote and zero information (and your graphing calculator) to draw the graph of f.
iv. Plot the horizontal line $y = k$ on your coordinate system and label this line with its equation.
v. Use your calculator's `intersect` utility to help determine the solution of $f(x) = k$. Label this point on your graph with its coordinates.
vi. Solve the equation $f(x) = k$ algebraically, placing the work for this soluton on your graph paper next to your coordinate system containing the graphical solution. Do the answers agree?

21. $f(x) = \dfrac{1}{x} + \dfrac{1}{x+5}, \quad k = 9/14$

22. $f(x) = \dfrac{1}{x} + \dfrac{1}{x-2}, \quad k = 8/15$

23. $f(x) = \dfrac{1}{x-1} - \dfrac{1}{x+1}, \quad k = 1/4$

24. $f(x) = \dfrac{1}{x-1} - \dfrac{1}{x+2}, \quad k = 1/6$

25. $f(x) = \dfrac{1}{x-2} + \dfrac{1}{x+2}, \quad k = 4$

26. $f(x) = \dfrac{1}{x-3} + \dfrac{1}{x+2}, \quad k = 5$

In **Exercises 27-34**, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

27. $\dfrac{2}{x+1} + \dfrac{4}{x+2} = -3$

28. $\dfrac{2}{x-5} - \dfrac{7}{x-7} = 9$

29. $\dfrac{3}{x+9} - \dfrac{2}{x+7} = -3$

30. $\dfrac{3}{x+9} - \dfrac{6}{x+7} = 9$

31. $\dfrac{2}{x+9} + \dfrac{2}{x+6} = -1$

32. $\dfrac{5}{x-6} - \dfrac{8}{x-7} = -1$

33. $\dfrac{3}{x+3} + \dfrac{6}{x+2} = -2$

34. $\dfrac{2}{x-4} - \dfrac{2}{x-1} = 1$

For each of the equations in **Exercises 35-40**, perform each of the following tasks.

i. Follow the lead of Example 10 in the text. Make one side of the equation equal to zero. Load the nonzero side into your calculator and draw its graph.
ii. Determine the vertical asymptotes of by analyzing the equation and the resulting graph on your calculator. Use the `TABLE` feature of your calculator to determine any horizontal asymptote behavior.
iii. Use the `zero` finding utility in the `CALC` menu to determine the zero of the nonzero side of the resulting equation.
iv. Set up a coordinate system on graph paper. Label and scale each axis. *Remember to draw all lines with a ruler.* Draw the graph of the nonzero side of the equation. Draw the vertical and horizontal asymptotes and label them with their equations. Plot the x-intercept and label it with its coordinates.
v. Use an algebraic technique to determine the solution of the equation and compare it with the solution found by the graphical analysis above.

35. $\dfrac{x}{x+1}+\dfrac{8}{x^2-2x-3}=\dfrac{2}{x-3}$

36. $\dfrac{x}{x+4}-\dfrac{2}{x+1}=\dfrac{12}{x^2+5x+4}$

37. $\dfrac{x}{x+1}-\dfrac{4}{2x+1}=\dfrac{2x-1}{2x^2+3x+2}$

38. $\dfrac{2x}{x-4}-\dfrac{1}{x+1}=\dfrac{4x+24}{x^2-3x-4}$

39. $\dfrac{x}{x-2}+\dfrac{3}{x+2}=\dfrac{8}{4-x^2}$

40. $\dfrac{x}{x-1}-\dfrac{4}{x+1}=\dfrac{x-6}{1-x^2}$

In **Exercises 41-68**, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

41. $\dfrac{x}{3x-9}-\dfrac{9}{x}=\dfrac{1}{x-3}$

42. $\dfrac{5x}{x+2}+\dfrac{5}{x-5}=\dfrac{x+6}{x^2-3x-10}$

43. $\dfrac{3x}{x+2}-\dfrac{7}{x}=-\dfrac{1}{2x+4}$

44. $\dfrac{4x}{x+6}-\dfrac{4}{x+4}=\dfrac{x-4}{x^2+10x+24}$

45. $\dfrac{x}{x-5}+\dfrac{9}{4-x}=\dfrac{x+5}{x^2-9x+20}$

46. $\dfrac{6x}{x-5}-\dfrac{2}{x-3}=\dfrac{x-8}{x^2-8x+15}$

47. $\dfrac{2x}{x-4}+\dfrac{5}{2-x}=\dfrac{x+8}{x^2-6x+8}$

48. $\dfrac{x}{x-7}-\dfrac{8}{5-x}=\dfrac{x+7}{x^2-12x+35}$

49. $-\dfrac{x}{2x+2}-\dfrac{6}{x}=-\dfrac{2}{x+1}$

50. $\dfrac{7x}{x+3}-\dfrac{4}{2-x}=\dfrac{x+8}{x^2+x-6}$

51. $\dfrac{2x}{x+5}-\dfrac{2}{6-x}=\dfrac{x-2}{x^2-x-30}$

52. $\dfrac{4x}{x+1}+\dfrac{6}{x+3}=\dfrac{x-9}{x^2+4x+3}$

53. $\dfrac{x}{x+7}-\dfrac{2}{x+5}=\dfrac{x+1}{x^2+12x+35}$

54. $\dfrac{5x}{6x+4}+\dfrac{6}{x}=\dfrac{1}{3x+2}$

55. $\dfrac{2x}{3x+9}-\dfrac{4}{x}=-\dfrac{2}{x+3}$

56. $\dfrac{7x}{x+1} - \dfrac{4}{x+2} = \dfrac{x+6}{x^2+3x+2}$

57. $\dfrac{x}{2x-8} + \dfrac{8}{x} = \dfrac{2}{x-4}$

58. $\dfrac{3x}{x-6} + \dfrac{6}{x-6} = \dfrac{x+2}{x^2-12x+36}$

59. $\dfrac{x}{x+2} + \dfrac{2}{x} = -\dfrac{5}{2x+4}$

60. $\dfrac{4x}{x-2} + \dfrac{2}{2-x} = \dfrac{x+4}{x^2-4x+4}$

61. $-\dfrac{2x}{3x-9} - \dfrac{3}{x} = -\dfrac{2}{x-3}$

62. $\dfrac{2x}{x+1} - \dfrac{2}{x} = \dfrac{1}{2x+2}$

63. $\dfrac{x}{x+1} + \dfrac{5}{x} = \dfrac{1}{4x+4}$

64. $\dfrac{2x}{x-4} - \dfrac{8}{x-7} = \dfrac{x+2}{x^2-11x+28}$

65. $-\dfrac{9x}{8x-2} + \dfrac{2}{x} = -\dfrac{2}{4x-1}$

66. $\dfrac{2x}{x-3} - \dfrac{4}{4-x} = \dfrac{x-9}{x^2-7x+12}$

67. $\dfrac{4x}{x+6} - \dfrac{5}{7-x} = \dfrac{x-5}{x^2-x-42}$

68. $\dfrac{x}{x-1} - \dfrac{4}{x} = \dfrac{1}{5x-5}$

7.7 *Solutions*

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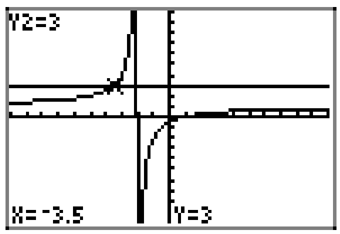
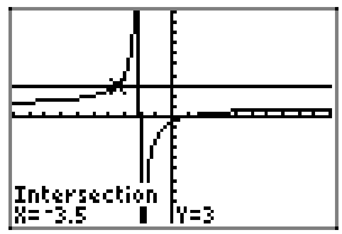
1. Load $f(x) = (x-1)/(x+2)$ into `Y1` and $y = 3$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the point of intersection, as shown in (b). *Note: We had some difficulty with this at first, because the calculator warned that it could not find a point of intersection. We tried a second time, but this time we made sure our "guess" was near the point of intersection (and certainly to the left of the vertical asymptote). This approach provided the solution in (b).*

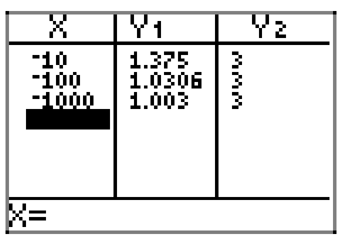


(a)

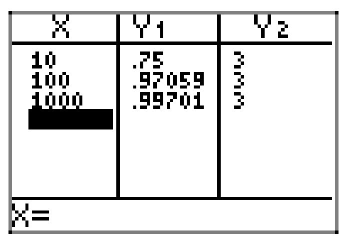


(b)

Note that $x = 1$ makes the numerator of $f(x) = (x-1)/(x+2)$ zero without making the denominator zero. Hence, $x = 1$ is a zero of f and $(1, 0)$ is an x-intercept of the graph of f. Secondly, the function f is reduced and $x = -2$ makes the denominator zero. Thus, $x = -2$ is a vertical asymptote. Tabular results indicate a horizontal asymptote $y = 1$.

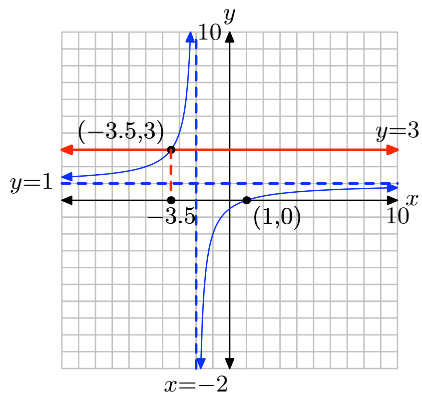


(c)



(d)

This is enough information to construct the graphs of $f(x) = (x-1)/(x+2)$ and $y = 3$ that follows. Note that we drop a dashed vertical line from the point of intersection to the x-axis where we label the x-value.



To solve the equation algebraically, first multiply both sides of the equation by $x + 2$.

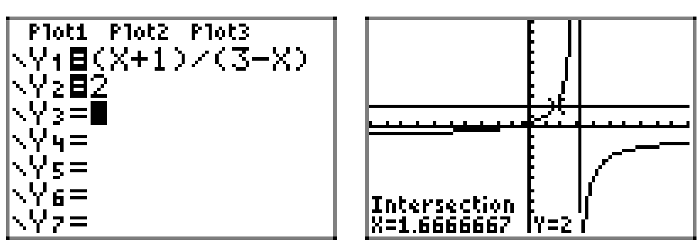
$$\begin{aligned} \frac{x-1}{x+2} &= 3 \\ (x+2)\left[\frac{x-1}{x+2}\right] &= 3(x+2) \\ x - 1 &= 3(x+2) \end{aligned}$$

Expand, then solve for x.

$$\begin{aligned} x - 1 &= 3x + 6 \\ x - 3x &= 6 + 1 \\ -2x &= 7 \\ x &= -7/2 \end{aligned}$$

Note how the solution $x = -7/2$ agrees with the graphical solution found above.

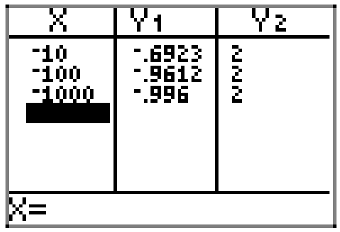
3. Load $f(x) = (x + 1)/(3 - x)$ into `Y1` and $y = 2$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the point of intersection, as shown in (b). *Note: We had some difficulty with this at first, because the calculator warned that it could not find a point of intersection. We tried a second time, but this time we made sure our "guess" was near the point of intersection (and certainly to the left of the vertical asymptote). This approach provided the solution in (b).*



(a) (b)

Note that $x = -1$ makes the numerator of $f(x) = (x+1)/(3-x)$ zero without making the denominator zero. Hence, $x = -1$ is a zero of f and $(-1, 0)$ is an x-intercept of the graph of f. Secondly, the function f is reduced and $x = 3$ makes the denominator zero.

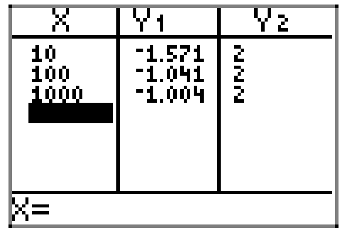
Thus, $x = 3$ is a vertical asymptote. Tabular results indicate a horizontal asymptote $y = -1$.



X	Y1	Y2
-10	-.6923	2
-100	-.9612	2
-1000	-.996	2

X=

(c)

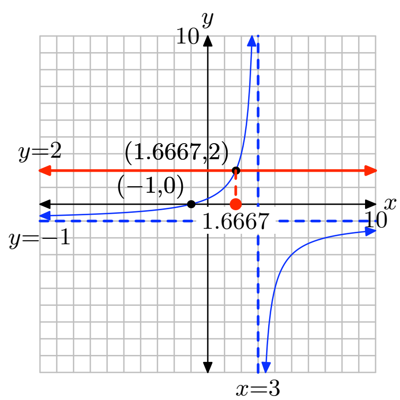


X	Y1	Y2
10	-1.571	2
100	-1.041	2
1000	-1.004	2

X=

(d)

This is enough information to construct the graphs of $f(x) = (x+1)/(3-x)$ and $y = 2$ that follows. Note that we drop a dashed vertical line from the point of intersection to the x-axis where we label the x-value.



To solve the equation algebraically, first multiply both sides of the equation by $3 - x$.

$$\frac{x+1}{3-x} = 2$$

$$(3-x)\left[\frac{x+1}{3-x}\right] = 2(3-x)$$

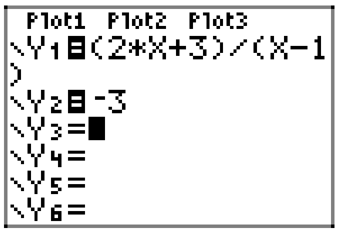
$$x + 1 = 2(3-x)$$

Expand, then solve for x.

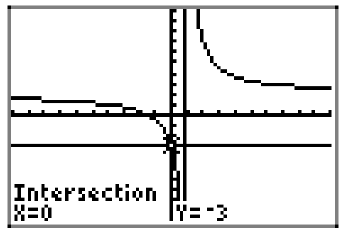
$$\begin{aligned} x + 1 &= 6 - 2x \\ x + 2x &= 6 - 1 \\ 3x &= 5 \\ x &= 5/3 \end{aligned}$$

Note how the solution $x = 5/3$ agrees with the graphical solution found above.

5. Load $f(x) = (2x+3)/(x-1)$ into `Y1` and $y = -3$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the point of intersection, as shown in (b). *Note: We had some difficulty with this at first, because the calculator warned that it could not find a point of intersection. We tried a second time, but this time we made sure our "guess" was near the point of intersection (and certainly to the left of the vertical asymptote). This approach provided the solution in (b).*

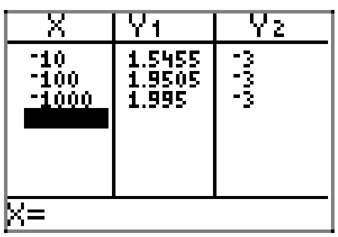


(a)



(b)

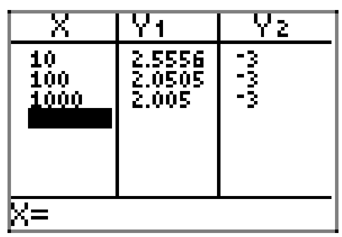
Note that $x = -3/2$ makes the numerator of $f(x) = (2x+3)/(x-1)$ zero without making the denominator zero. Hence, $x = -3/2$ is a zero of f and $(-3/2, 0)$ is an x-intercept of the graph of f. Secondly, the function f is reduced and $x = 1$ makes the denominator zero. Thus, $x = 1$ is a vertical asymptote. Tabular results indicate a horizontal asymptote $y = 2$.



X	Y1	Y2
-10	1.5455	-3
-100	1.9505	-3
-1000	1.995	-3

X=

(c)

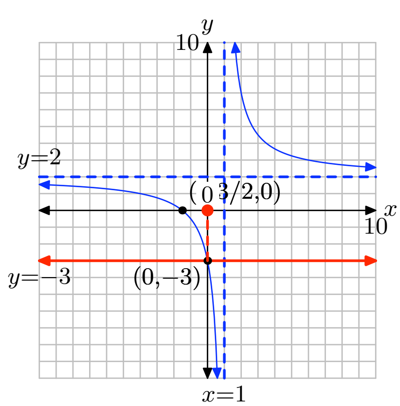


X	Y1	Y2
10	2.5556	-3
100	2.0505	-3
1000	2.005	-3

X=

(d)

This is enough information to construct the graphs of $f(x) = (2x+3)/(x-1)$ and $y = -3$ that follows. Note that we drop a dashed vertical line from the point of intersection to the x-axis where we label the x-value.



To solve the equation algebraically, first multiply both sides of the equation by $x - 1$.

$$\begin{aligned} \frac{2x+3}{x-1} &= -3 \\ (x-1)\left[\frac{2x+3}{x-1}\right] &= -3(x-1) \\ 2x+3 &= -3(x-1) \end{aligned}$$

Expand, then solve for x.

$$\begin{aligned} 2x+3 &= -3x+3 \\ 2x+3x &= 3-3 \\ 5x &= 0 \\ x &= 0 \end{aligned}$$

Note how the solution $x = 0$ agrees with the graphical solution found above.

7. To solve $\frac{16x-9}{2x-1} = 8$, first clear fractions by multiplying both sides of the equation by the LCD $2x - 1$ to obtain

$$16x - 9 = 16x - 8$$

Then solve this linear equation. In this case, there are no solutions.

9. To solve $\frac{5x+8}{4x+1} = -11$, first clear fractions by multiplying both sides of the equation by the LCD $4x + 1$ to obtain

$$5x + 8 = -44x - 11$$

Then solve this linear equation to obtain the solution $-\frac{19}{49}$. Since this value does not cause division by zero in the original equation, it is a valid solution.

11. To solve $-\frac{35x}{7x+12} = -5$, first clear fractions by multiplying both sides of the equation by the LCD $7x + 12$ to obtain

$$-35x = -35x - 60$$

Then solve this linear equation. In this case, there are no solutions.

13. To solve $\frac{8x+2}{x-11} = 11$, first clear fractions by multiplying both sides of the equation by the LCD $x - 11$ to obtain

$$8x + 2 = 11x - 121$$

Then solve this linear equation to obtain the solution 41. Since this value does not cause division by zero in the original equation, it is a valid solution.

15. The least common denominator (LCD) is 63, so first clear fractions by multiplying both sides of the equation by the LCD to obtain

$$9x + 56 = -72$$

Then solve this linear equation to obtain the solution $-\frac{128}{9}$. Since this value does not cause division by zero in the original equation, it is a valid solution.

17. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the quadratic equation

$$-57x = 27x^2 - 40$$

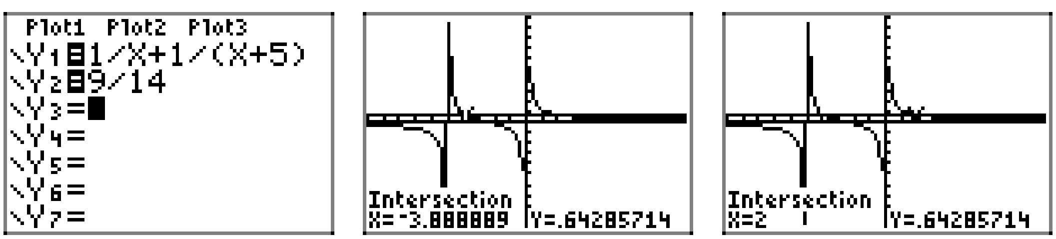
Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions $-\frac{8}{3}$ and $\frac{5}{9}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

19. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the quadratic equation

$$7x = 4x^2 - 3$$

Then solve this quadratic equation by using the quadratic formula to obtain the solutions $\frac{7+\sqrt{97}}{8}$ and $\frac{7-\sqrt{97}}{8}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

21. Load $f(x) = 1/x + 1/(x+5)$ into `Y1` and $y = 9/14$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the points of intersection, as shown in (b) and (c).

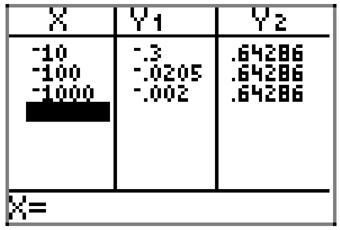


(a) (b) (c)

We can combine terms of f.

$$f(x) = \frac{1}{x} + \frac{1}{x+5} = \frac{x+5}{x(x+5)} + \frac{x}{x(x+5)} = \frac{2x+5}{x(x+5)}$$

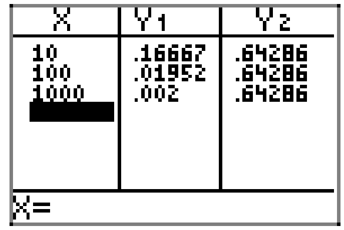
Note that $x = -5/2$ makes the numerator of $f(x) = (2x+5)/(x(x+5))$ zero without making the denominator zero. Hence, $x = -5/2$ is a zero of f and $(-5/2, 0)$ is an x-intercept of the graph of f. Secondly, the function f is reduced and $x = -5$ and $x = 0$ make the denominator zero. Thus, $x = -5$ and $x = 0$ are vertical asymptotes. Tabular results indicate a horizontal asymptote $y = 0$.



X	Y1	Y2
-10	-.3	.64286
-100	-.0205	.64286
-1000	-.002	.64286

X=

(d)

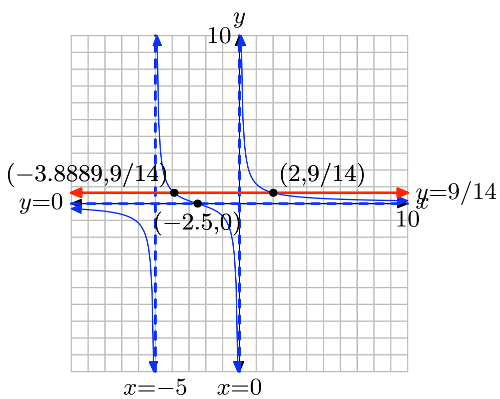


X	Y1	Y2
10	.16667	.64286
100	.01952	.64286
1000	.002	.64286

X=

(e)

This is enough information to construct the graphs of $f(x) = (2x+5)/(x(x+5))$ and $y = 9/14$ that follows. The x-value of each point of intersection is a solution of the equation $f(x) = 9/14$. *Note: In this instance, we avoided the usual dashed vertical lines because we thought the final picture would be too crowded.*



To solve the equation algebraically, first multiply both sides of the equation by $14x(x+5)$.

$$\frac{2x+5}{x(x+5)} = \frac{9}{14}$$

$$14x(x+5)\left[\frac{2x+5}{x(x+5)}\right] = \frac{9}{14}14x(x+5)$$

$$14(2x+5) = 9x(x+5)$$

Expand, make one side zero.

$$28x + 70 = 9x^2 + 45x$$

$$0 = 9x^2 + 17x - 70$$

Let's use the quadratic formula.

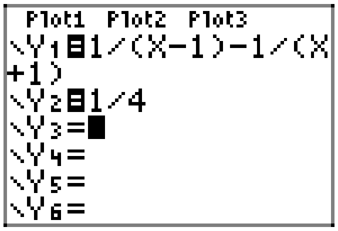
$$x = \frac{-17 \pm \sqrt{17^2 - 4(9)(-70)}}{2(9)} = \frac{-17 \pm \sqrt{2809}}{18} = \frac{-17 \pm 53}{18}$$

This gives us two answers,

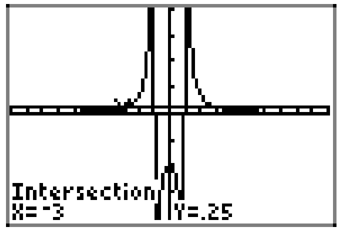
$$x = \frac{-17-53}{18} = -\frac{35}{9} \quad \text{and} \quad x = \frac{-17+53}{18} = 2.$$

Note how these solutions agree with the graphical solutions found above.

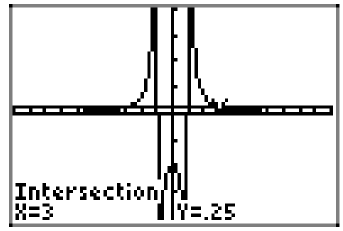
23. Load $f(x) = 1/(x-1) - 1/(x+1)$ into `Y1` and $y = 1/4$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the points of intersection, as shown in (b) and (c).



(a)



(b)

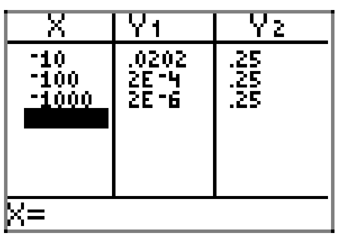


(c)

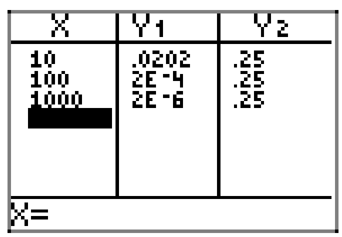
We can combine terms of f.

$$f(x) = \frac{1}{x-1} - \frac{1}{x+1} = \frac{x+1}{(x-1)(x+1)} - \frac{x-1}{(x-1)(x+1)} = \frac{2}{(x-1)(x+1)}$$

There is no value of x that will make the numerator zero. Hence, the graph of f has no x-intercept. Secondly, the function f is reduced and $x = -1$ and $x = 1$ make the denominator zero. Thus, $x = -1$ and $x = 1$ are vertical asymptotes. Tabular results indicate a horizontal asymptote $y = 0$.

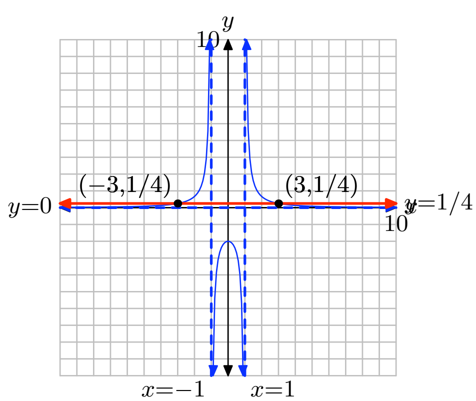


(d)



(e)

This is enough information to construct the graphs of $f(x) = 2/((x-1)(x+1))$ and $y = 1/4$ that follows. The x-value of each point of intersection is a solution of the equation $f(x) = 1/4$. *Note: In this instance, we avoided the usual dashed vertical lines because we thought the final picture would be too crowded.*

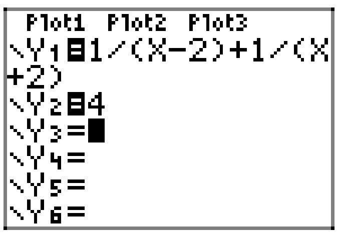


To solve the equation algebraically, first multiply both sides of the equation by $4(x-1)(x+1)$.

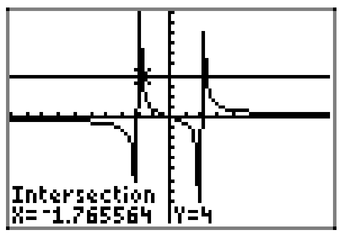
$$\begin{aligned}
\frac{2}{(x-1)(x+1)} &= \frac{1}{4} \\
4(x-1)(x+1)\left[\frac{2}{(x-1)(x+1)}\right] &= \frac{1}{4}4(x-1)(x+1) \\
8 &= x^2-1 \\
9 &= x^2 \\
x &= \pm 3
\end{aligned}$$

Note how these solutions agree with the graphical solutions found above.

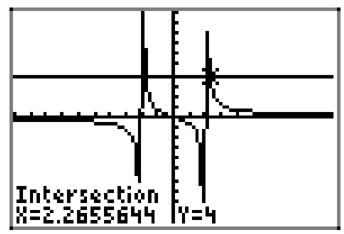
25. Load $f(x) = 1/(x-2) + 1/(x+2)$ into `Y1` and $y = 4$ in `Y2`, as shown in (a). Use the `intersect` utility from the `CALC` menu to find the points of intersection, as shown in (b) and (c).



(a)



(b)

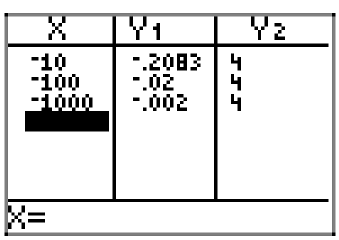


(c)

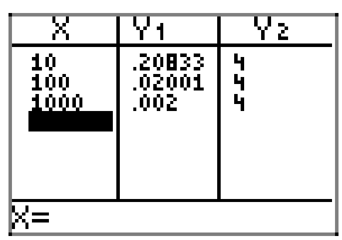
We can combine terms of f.

$$f(x) = \frac{1}{x-2} + \frac{1}{x+2} = \frac{x+2}{(x-2)(x+2)} + \frac{x-2}{(x-2)(x+2)} = \frac{2x}{(x-2)(x+2)}$$

Note that $x = 0$ makes the numerator equal to zero without making the denominator zero. Hence, $x = 0$ is a zero of f and $(0, 0)$ is an x-intercept of the graph of f. Secondly, the function f is reduced and $x = -2$ and $x = 2$ make the denominator zero. Thus, $x = -2$ and $x = 2$ are vertical asymptotes. Tabular results indicate a horizontal asymptote $y = 0$.



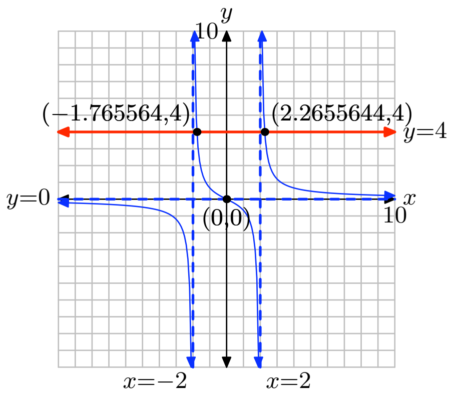
(d)



(e)

This is enough information to construct the graphs of $f(x) = 2x/((x-2)(x+2))$ and $y = 4$ that follows. The x-value of each point of intersection is a solution of the equation

$f(x) = 4$. *Note: In this instance, we avoided the usual dashed vertical lines because we thought the final picture would be too crowded.*



To solve the equation algebraically, first multiply both sides of the equation by $(x-2)(x+2)$.

$$\begin{aligned} \frac{2x}{(x-2)(x+2)} &= 4 \\ (x-2)(x+2)\left[\frac{2x}{(x-2)(x+2)}\right] &= 4(x-2)(x+2) \\ 2x &= 4(x^2-4) \\ 2x &= 4x^2-16 \end{aligned}$$

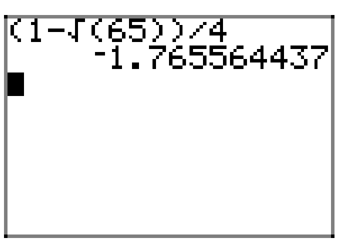
Make one side zero and divide both side of the resulting equation by 2.

$$\begin{aligned} 0 &= 4x^2-2x-16 \\ 0 &= 2x^2-x-8 \end{aligned}$$

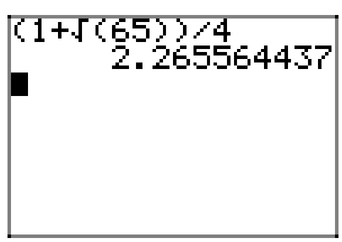
Let's use the quadratic formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)} = \frac{1 \pm \sqrt{65}}{4}$$

If you approximate these with your calculator, you will see that they match the approximations found graphically above.



(f)



(g)

Note how these solutions agree with the graphical solutions found above.

27. The least common denominator (LCD) is $(x+1)(x+2)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$2(x+2)+4(x+1)=-3(x+1)(x+2)$$

Then simplify this quadratic equation to standard form:

$$-3x^2-15x-14=0$$

Finally, solve this quadratic equation by using the quadratic formula to obtain the solutions $\frac{-15+\sqrt{57}}{6}$ and $\frac{-15-\sqrt{57}}{6}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

29. The least common denominator (LCD) is $(x+9)(x+7)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$3(x+7)-2(x+9)=-3(x+9)(x+7)$$

Then simplify this quadratic equation to standard form:

$$-3x^2-49x-192=0$$

Finally, solve this quadratic equation by using the quadratic formula to obtain the solutions $\frac{-49+\sqrt{97}}{6}$ and $\frac{-49-\sqrt{97}}{6}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

31. The least common denominator (LCD) is $(x+9)(x+6)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$2(x+6)+2(x+9)=-1(x+9)(x+6)$$

Then simplify this quadratic equation to standard form:

$$-x^2-19x-84=0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions -7 and -12. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

33. The least common denominator (LCD) is $(x+3)(x+2)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$3(x+2)+6(x+3)=-2(x+3)(x+2)$$

Then simplify this quadratic equation to standard form:

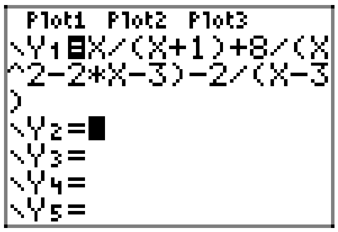
$$-2x^2-19x-36=0$$

Finally, solve this quadratic equation by using the quadratic formula to obtain the solutions $\frac{-19+\sqrt{73}}{4}$ and $\frac{-19-\sqrt{73}}{4}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

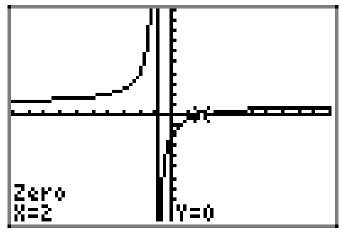
35. Set

$$\frac{x}{x+1} + \frac{8}{x^2 - 2x - 3} - \frac{2}{x-3} = 0,$$

and load the left-hand side into `Y1`, as shown in (a). Use the `zero` finding utility in the `CALC` menu to determine the zero, as shown in (b).

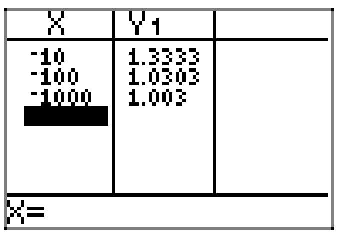


(a)



(b)

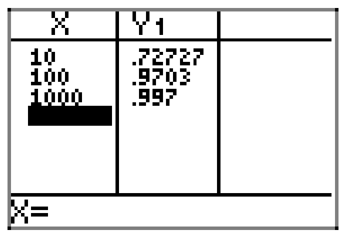
The equation would indicate vertical asymptotes at $x = -1$ and $x = 3$, but the image in (b) would indicate that some sort of cancellation takes place, leaving only one vertical asymptote at $x = -1$. The following tables indicate the presence of a horizontal asymptote $y = 1$.



X	Y1	
-10	1.3333	
-100	1.0303	
-1000	1.003	

X=

(c)

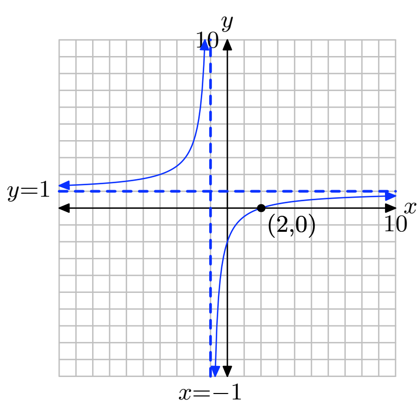


X	Y1	
10	.72727	
100	.9703	
1000	.997	

X=

(d)

Copy the image onto your homework as follows.



To solve the equation algebraically, multiply both sides of the equation by the common denominator $(x+1)(x-3)$.

$$
\begin{aligned}
\frac{x}{x+1} + \frac{8}{x^2-2x-3} &= \frac{2}{x-3}\\
(x+1)(x-3)\left[\frac{x}{x+1} + \frac{8}{(x+1)(x-3)}\right] &= \left[\frac{2}{x-3}\right](x+1)(x-3)\\
x(x-3)+8 &= 2(x+1)
\end{aligned}
$$

Expand, then make one side zero and factor.

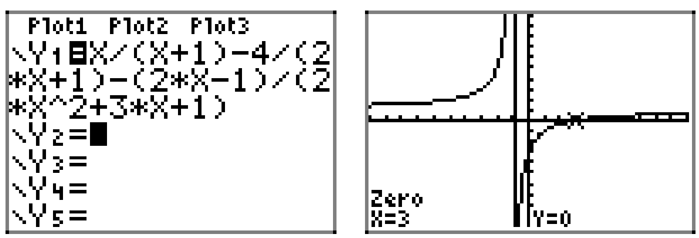
$$
\begin{aligned}
x^2-3x+8 &= 2x+2\\
x^2-5x+6 &= 0\\
(x-3)(x-2) &= 0
\end{aligned}
$$

This would indicate two solutions. However, note that $x = 3$ is an extraerroeous answer, as it makes some rational expressions in the original equation undefined. Hence, $x = 2$ is the only solution. Note that this agrees with the graphical solution above.

37. Set

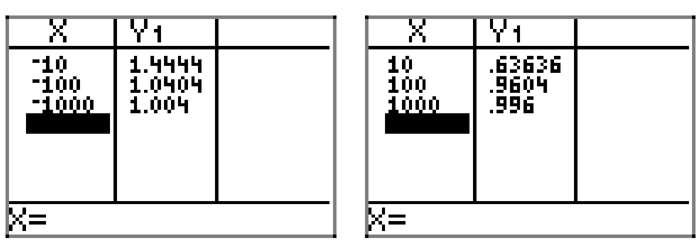
$$\frac{x}{x+1} - \frac{4}{2x+1} - \frac{2x-1}{2x^2+3x+1} = 0$$

and load the left-hand side into `Y1`, as shown in (a). Use the `zero` finding utility in the `CALC` menu to determine the zero, as shown in (b).



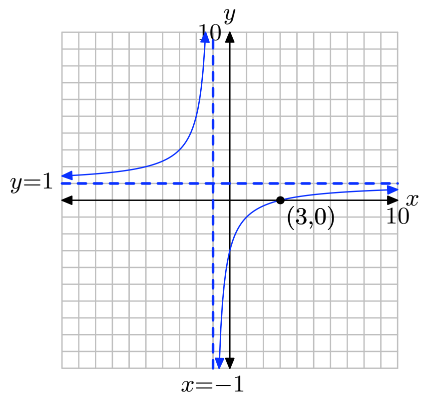
(a) (b)

The equation would indicate vertical asymptotes at $x = -1$ and $x = -1/2$, but the image in (b) would indicate that some sort of cancellation takes place, leaving only one vertical asymptote at $x = -1$. The following tables indicate the presence of a horizontal asymptote $y = 1$.



(c) (d)

Copy the image onto your homework as follows.



To solve the equation algebraically, multiply both sides of the equation by the common denominator $(x+1)(2x+1)$.

$$\frac{x}{x+1} - \frac{4}{2x+1} = \frac{2x-1}{2x^2+3x+1}$$
$$(x+1)(2x+1)\left[\frac{x}{x+1} - \frac{4}{2x+1}\right] = \left[\frac{2x-1}{(2x+1)(x+1)}\right](x+1)(2x+1)$$
$$x(2x+1) - 4(x+1) = 2x-1$$

Expand, simplify, then make one side zero and factor.

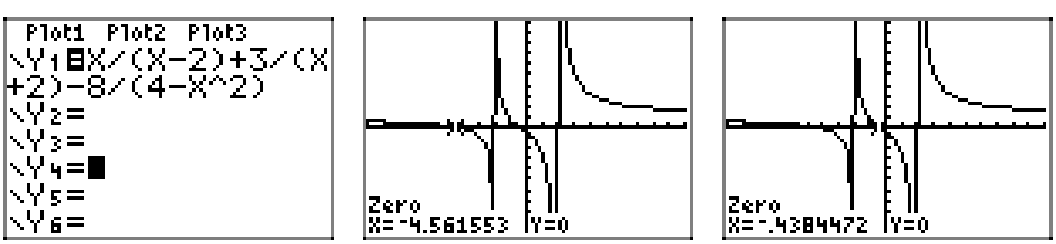
$$2x^2 + x - 4x - 4 = 2x - 1$$
$$2x^2 - 3x - 4 = 2x - 1$$
$$2x^2 - 5x - 3 = 0$$
$$(2x+1)(x-3) = 0$$

This would indicate two solutions. However, note that $x = -1/2$ is an extraerroeous answer, as it makes some rational expressions in the original equation undefined. Hence, $x = 3$ is the only solution. Note that this agrees with the graphical solution above.

39. Set

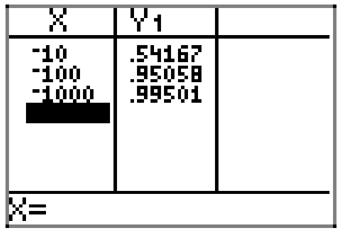
$$\frac{x}{x-2} + \frac{3}{x+2} - \frac{8}{4-x^2} = 0$$

and load the left-hand side into `Y1`, as shown in (a). Use the `zero` finding utility in the `CALC` menu to determine the zero, as shown in (b).



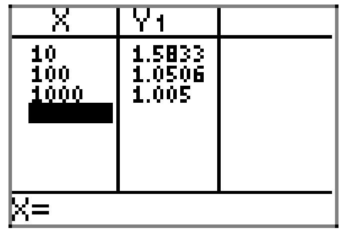
(a) (b) (c)

The equation and graph indicate vertical asymptotes at $x = -2$ and $x = 2$. The following tables indicate the presence of a horizontal asymptote $y = 1$.



X	Y1
-10	.54167
-100	.95058
-1000	.99501

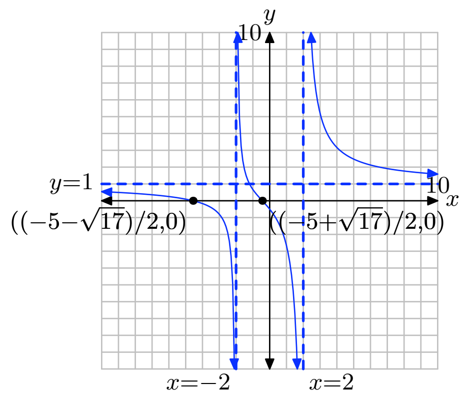
(d)



X	Y1
10	1.5833
100	1.0506
1000	1.005

(e)

Copy the image onto your homework as follows.



First, make a sign change, negating two parts of the last rational expression on the right-hand side of the equation.

$$\frac{x}{x-2} + \frac{3}{x+2} = \frac{-8}{x^2-4} \tag{1}$$

To solve the equation algebraically, multiply both sides of the equation by the common denominator $(x+2)(x-2)$.

$$\begin{aligned} \frac{x}{x-2} + \frac{3}{x+2} &= \frac{-8}{(x+2)(x-2)} \\ (x+2)(x-2)\left[\frac{x}{x-2} + \frac{3}{x+2}\right] &= \left[\frac{-8}{(x+2)(x-2)}\right](x+2)(x-2) \\ x(x+2) + 3(x-2) &= -8 \end{aligned}$$

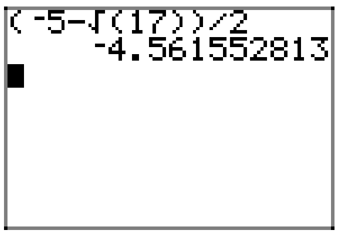
Expand, simplify, then make one side zero and factor.

$$\begin{aligned} x^2 + 2x + 3x - 6 &= -8 \\ x^2 + 5x - 6 &= -8 \\ x^2 - 5x + 2 &= 0 \end{aligned}$$

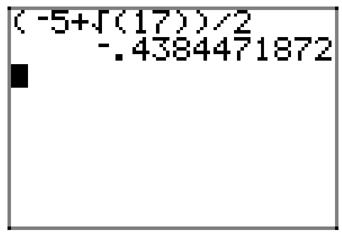
Let's use the quadratic formula.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)} = \frac{-5 \pm \sqrt{17}}{2}$$

To compare these exact solutions to the approximate solutions found by graphical analysis above, use your calculator to find decimal approximations, as shown in (f) and (g) below.



(f)



(g)

41. The least common denominator (LCD) is $x(3x - 9)$, so multiply both sides of the equation by the LCD to obtain

$$x^2 - 9(3x - 9) = 3x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 27 and 3. However, 3 causes division by zero when substituted for x in the original equation. Therefore, 27 is the only valid solution.

43. The least common denominator (LCD) is $x(2x+4)$, so multiply both sides of the equation by the LCD to obtain

$$6x^2 - 7(2x + 4) = -x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions $\frac{7}{2}$ and $-\frac{4}{3}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

45. The least common denominator (LCD) is $(x-5)(x-4)$. However, the denominator of the second term is $4 - x$, so first rewrite the left side by negating the numerator and denominator of the second term:

$$\frac{x}{x-5} - \frac{9}{x-4} = \frac{x+5}{x^2 - 9x + 20}$$

Then clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$x(x - 4) - 9(x - 5) = x + 5$$

Then simplify this quadratic equation to standard form:

$$x^2 - 14x + 40 = 0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 10 and 4. However, 4 causes division by zero when substituted for x in the original equation. Therefore, 10 is the only valid solution.

47. The least common denominator (LCD) is $(x-4)(x-2)$. However, the denominator of the second term is $2-x$, so first rewrite the left side by negating the numerator and denominator of the second term:

$$\frac{2x}{x-4} - \frac{5}{x-2} = \frac{x+8}{x^2-6x+8}$$

Then clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$2x(x-2) - 5(x-4) = x+8$$

Then simplify this quadratic equation to standard form:

$$2x^2 - 10x + 12 = 0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 3 and 2. However, 2 causes division by zero when substituted for x in the original equation. Therefore, 3 is the only valid solution.

49. The least common denominator (LCD) is $x(2x+2)$, so multiply both sides of the equation by the LCD to obtain

$$-x^2 - 6(2x+2) = -4x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions -6 and -2. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

51. The least common denominator (LCD) is $(x+5)(x-6)$. However, the denominator of the second term is $6-x$, so first rewrite the left side by negating the numerator and denominator of the second term:

$$\frac{2x}{x+5} + \frac{2}{x-6} = \frac{x-2}{x^2-x-30}$$

Then clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$2x(x-6) + 2(x+5) = x-2$$

Then simplify this quadratic equation to standard form:

$$2x^2 - 11x + 12 = 0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 4 and $\frac{3}{2}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

53. The least common denominator (LCD) is $(x+7)(x+5)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$x(x+5) - 2(x+7) = x+1$$

Then simplify this quadratic equation to standard form:

$$x^2 + 2x - 15 = 0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 3 and -5. However, -5 causes division by zero when substituted for x in the original equation. Therefore, 3 is the only valid solution.

55. The least common denominator (LCD) is $x(3x+9)$, so multiply both sides of the equation by the LCD to obtain

$$2x^2 - 4(3x+9) = -6x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 6 and -3. However, -3 causes division by zero when substituted for x in the original equation. Therefore, 6 is the only valid solution.

57. The least common denominator (LCD) is $x(2x-8)$, so multiply both sides of the equation by the LCD to obtain

$$x^2 + 8(2x-8) = 4x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions 4 and -16. However, 4 causes division by zero when substituted for x in the original equation. Therefore, -16 is the only valid solution.

59. The least common denominator (LCD) is $x(2x+4)$, so multiply both sides of the equation by the LCD to obtain

$$2x^2 + 2(2x+4) = -5x$$

Then solve this quadratic equation by using the quadratic formula to obtain the solutions

$$\frac{-9+\sqrt{17}}{4} \quad \text{and} \quad \frac{-9-\sqrt{17}}{4}$$

Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

61. The least common denominator (LCD) is $x(3x-9)$, so multiply both sides of the equation by the LCD to obtain

$$-2x^2 - 3(3x-9) = -6x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions $-\frac{9}{2}$ and 3. However, 3 causes division by zero when substituted for x in the original equation. Therefore, $-\frac{9}{2}$ is the only valid solution.

63. The least common denominator (LCD) is $x(4x+4)$, so multiply both sides of the equation by the LCD to obtain

$$4x^2 + 5(4x+4) = x$$

Then solve this quadratic equation by using the quadratic formula to obtain the solutions

$$\frac{-19+\sqrt{41}}{8} \quad \text{and} \quad \frac{-19-\sqrt{41}}{8}$$

Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

65. The least common denominator (LCD) is $x(8x-2)$, so multiply both sides of the equation by the LCD to obtain

$$-9x^2 + 2(8x-2) = -4x$$

Then solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions $\frac{2}{9}$ and 2. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

67. The least common denominator (LCD) is $(x+6)(x-7)$. However, the denominator of the second term is $7-x$, so first rewrite the left side by negating the numerator and denominator of the second term:

$$\frac{4x}{x+6} + \frac{5}{x-7} = \frac{x-5}{x^2-x-42}$$

Then clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$4x(x-7) + 5(x+6) = x-5$$

Then simplify this quadratic equation to standard form:

$$4x^2 - 24x + 35 = 0$$

Finally, solve this quadratic equation either by factoring or by using the quadratic formula to obtain the solutions $\frac{7}{2}$ and $\frac{5}{2}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

7.8 Exercises

1. The sum of the reciprocals of two consecutive odd integers is $-\frac{16}{63}$. Find the two numbers.

2. The sum of the reciprocals of two consecutive odd integers is $\frac{28}{195}$. Find the two numbers.

3. The sum of the reciprocals of two consecutive integers is $-\frac{19}{90}$. Find the two numbers.

4. The sum of a number and its reciprocal is $\frac{41}{20}$. Find the number(s).

5. The sum of the reciprocals of two consecutive even integers is $\frac{5}{12}$. Find the two numbers.

6. The sum of the reciprocals of two consecutive integers is $\frac{19}{90}$. Find the two numbers.

7. The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number(s).

8. The sum of a number and its reciprocal is $\frac{5}{2}$. Find the number(s).

9. The sum of the reciprocals of two consecutive even integers is $\frac{11}{60}$. Find the two numbers.

10. The sum of a number and twice its reciprocal is $\frac{17}{6}$. Find the number(s).

11. The sum of the reciprocals of two numbers is 15/8, and the second number is 2 larger than the first. Find the two numbers.

12. The sum of the reciprocals of two numbers is 16/15, and the second number is 1 larger than the first. Find the two numbers.

---

13. Moira can paddle her kayak at a speed of 2 mph in still water. She paddles 3 miles upstream against the current and then returns to the starting location. The total time of the trip is 9 hours. What is the speed (in mph) of the current? Round your answer to the nearest hundredth.

14. Boris is kayaking in a river with a 6 mph current. Suppose that he can kayak 4 miles upstream in the same amount of time as it takes him to kayak 9 miles downstream. Find the speed (mph) of Boris's kayak in still water.

15. Jacob can paddle his kayak at a speed of 6 mph in still water. He paddles 5 miles upstream against the current and then returns to the starting location. The total time of the trip is 5 hours. What is the speed (in mph) of the current? Round your answer to the nearest hundredth.

16. Boris can paddle his kayak at a speed of 6 mph in still water. If he can paddle 5 miles upstream in the same amount of time as it takes his to paddle 9 miles downstream, what is the speed of the current?

17. Jacob is canoeing in a river with a 5 mph current. Suppose that he can canoe 4 miles upstream in the same amount of time as it takes him to canoe 8 miles downstream. Find the speed (mph) of Jacob's canoe in still water.

18. The speed of a freight train is 16 mph slower than the speed of a passenger train. The passenger train travels 518 miles in the same time that the freight train travels 406 miles. Find the speed of the freight train.

19. The speed of a freight train is 20 mph slower than the speed of a passenger train. The passenger train travels 440 miles in the same time that the freight train travels 280 miles. Find the speed of the freight train.

20. Emily can paddle her canoe at a speed of 2 mph in still water. She paddles 5 miles upstream against the current and then returns to the starting location. The total time of the trip is 6 hours. What is the speed (in mph) of the current? Round your answer to the nearest hundredth.

21. Jacob is canoeing in a river with a 2 mph current. Suppose that he can canoe 2 miles upstream in the same amount of time as it takes him to canoe 5 miles downstream. Find the speed (mph) of Jacob's canoe in still water.

22. Moira can paddle her kayak at a speed of 2 mph in still water. If she can paddle 4 miles upstream in the same amount of time as it takes her to paddle 8 miles downstream, what is the speed of the current?

23. Boris can paddle his kayak at a speed of 6 mph in still water. If he can paddle 5 miles upstream in the same amount of time as it takes his to paddle 10 miles downstream, what is the speed of the current?

24. The speed of a freight train is 19 mph slower than the speed of a passenger train. The passenger train travels 544 miles in the same time that the freight train travels 392 miles. Find the speed of the freight train.

---

25. It takes Jean 15 hours longer to complete an inventory report than it takes Sanjay. If they work together, it takes them 10 hours. How many hours would it take Sanjay if he worked alone?

26. Jean can paint a room in 5 hours. It takes Amelie 10 hours to paint the same room. How many hours will it take if they work together?

27. It takes Amelie 18 hours longer to complete an inventory report than it takes Jean. If they work together, it takes them 12 hours. How many hours would it take Jean if she worked alone?

28. Sanjay can paint a room in 5 hours. It takes Amelie 9 hours to paint the same room. How many hours will it take if they work together?

29. It takes Ricardo 12 hours longer to complete an inventory report than it takes Sanjay. If they work together, it takes them 8 hours. How many hours would it take Sanjay if he worked alone?

30. It takes Ricardo 8 hours longer to complete an inventory report than it takes Amelie. If they work together, it takes them 3 hours. How many hours would it take Amelie if she worked alone?

31. Jean can paint a room in 4 hours. It takes Sanjay 7 hours to paint the same room. How many hours will it take if they work together?

32. Amelie can paint a room in 5 hours. It takes Sanjay 9 hours to paint the same room. How many hours will it take if they work together?

7.8 *Solutions*

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1. Start with the equation

$$\frac{1}{x} + \frac{1}{x+2} = -\frac{16}{63}$$

Multiply both sides by the LCD $63x(x+2)$:

$$63(x+2) + 63x = -16x(x+2)$$

Simplify to obtain

$$126x + 126 = -16x^2 - 32x$$

This quadratic equation has one integer solution $x = -9$, and the second number is $-9 + 2 = -7$.

3. Start with the equation

$$\frac{1}{x} + \frac{1}{x+1} = -\frac{19}{90}$$

Multiply both sides by the LCD $90x(x+1)$:

$$90(x+1) + 90x = -19x(x+1)$$

Simplify to obtain

$$180x + 90 = -19x^2 - 19x$$

This quadratic equation has one integer solution $x = -10$, and the second number is $-10 + 1 = -9$.

5. Start with the equation

$$\frac{1}{x} + \frac{1}{x+2} = \frac{5}{12}$$

Multiply both sides by the LCD $12x(x+2)$:

$$12(x+2) + 12x = 5x(x+2)$$

Simplify to obtain

$$24x + 24 = 5x^2 + 10x$$

This quadratic equation has one integer solution $x = 4$, and the second number is $4 + 2 = 6$.

7. Start with the equation

$$x + 2\left(\frac{1}{x}\right) = \frac{9}{2}$$

Multiply both sides by the LCD $2x$:

$$2x^2 + 4 = 9x$$

Solve this quadratic equation to get $x = \frac{1}{2}, 4$.

9. Start with the equation

$$\frac{1}{x} + \frac{1}{x+2} = \frac{11}{60}$$

Multiply both sides by the LCD $60x(x+2)$:

$$60(x+2) + 60x = 11x(x+2)$$

Simplify to obtain

$$120x + 120 = 11x^2 + 22x$$

This quadratic equation has one integer solution $x = 10$, and the second number is $10 + 2 = 12$.

11. Start with the equation

$$\frac{1}{x} + \frac{1}{x+2} = \frac{15}{8}$$

Multiply both sides by the LCD $8x(x+2)$:

$$8(x+2) + 8x = 15x(x+2)$$

Simplify to obtain

$$16x + 16 = 15x^2 + 30x$$

Solve this quadratic equation to get $x = \frac{2}{3}, \frac{-8}{5}$. For $x = \frac{2}{3}$, the other number is $x + 2 = \frac{2}{3} + 2 = \frac{8}{3}$; and, for $x = \frac{-8}{5}$, the other number is $x + 2 = \frac{-8}{5} + 2 = \frac{2}{5}$. Thus, the possible pairs of numbers are $\{\frac{2}{3}, \frac{8}{3}\}$ and $\{\frac{-8}{5}, \frac{2}{5}\}$.

13. Let t_1 represent the time it takes for the upstream part of the trip, and t_2 represent the time it takes for the downstream part of the trip. Thus,

$$t_1 + t_2 = 9 \text{ hours}$$

Also, let c represent the speed of the current. Then the actual speed of the boat going upstream is $2 - c$, and the speed downstream is $2 + c$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\begin{aligned} \text{Trip upstream: } 3 &= (2 - c)t_1 \\ \text{Trip downstream: } 3 &= (2 + c)t_2 \end{aligned}$$

Solving for the time variable in each equation, it follows that

$$t_1 = \frac{3}{2 - c} \qquad \text{and} \qquad t_2 = \frac{3}{2 + c}$$

Therefore,

$$9 = t_1 + t_2 = \frac{3}{2 - c} + \frac{3}{2 + c}$$

Now solve this rational equation for c:

$$\begin{aligned} 9 = \frac{3}{2 - c} + \frac{3}{2 + c} &\implies 9(2 - c)(2 + c) = 3(2 + c) + 3(2 - c) \\ &\implies 36 - 9c^2 = 12 \\ &\implies 9c^2 = 24 \\ &\implies c = \pm\sqrt{\frac{8}{3}} \end{aligned}$$

Discarding the negative answer, the speed of the current is ≈ 1.63 mph.

15. Let t_1 represent the time it takes for the upstream part of the trip, and t_2 represent the time it takes for the downstream part of the trip. Thus,

$$t_1 + t_2 = 5 \text{ hours}$$

Also, let c represent the speed of the current. Then the actual speed of the boat going upstream is $6 - c$, and the speed downstream is $6 + c$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\begin{aligned} \text{Trip upstream: } 5 &= (6 - c)t_1 \\ \text{Trip downstream: } 5 &= (6 + c)t_2 \end{aligned}$$

Solving for the time variable in each equation, it follows that

$$t_1 = \frac{5}{6-c} \qquad \text{and} \qquad t_2 = \frac{5}{6+c}$$

Therefore,

$$5 = t_1 + t_2 = \frac{5}{6-c} + \frac{5}{6+c}$$

Now solve this rational equation for c:

$$\begin{aligned} 5 = \frac{5}{6-c} + \frac{5}{6+c} &\implies 5(6-c)(6+c) = 5(6+c) + 5(6-c) \\ &\implies 180 - 5c^2 = 60 \\ &\implies 5c^2 = 120 \\ &\implies c = \pm\sqrt{24} \end{aligned}$$

Discarding the negative answer, the speed of the current is ≈ 4.90 mph.

17. Let t represent the time it takes for each part of the trip, and let r represent the speed of the boat in still water. Then the actual speed of the boat going downstream is $r + 5$, and the speed upstream is $r - 5$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\begin{aligned} \text{Trip downstream: } 8 &= (r+5)t \\ \text{Trip upstream: } 4 &= (r-5)t \end{aligned}$$

Solving for t in each equation, it follows that

$$\frac{8}{r+5} = t = \frac{4}{r-5}$$

Now solve this rational equation for r:

$$\begin{aligned} \frac{8}{r+5} = \frac{4}{r-5} &\implies 8(r-5) = 4(r+5) \\ &\implies 4r = 60 \\ &\implies r = 15 \text{ mph} \end{aligned}$$

19. Let t represent the time it takes for each trip, and let r represent the speed of the freight train. Then the speed of the passenger train is $r + 20$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\text{Freight train: } 280 = rt$$
$$\text{Passenger train: } 440 = (r + 20)t$$

Solving for t in each equation, it follows that

$$\frac{280}{r} = t = \frac{440}{r + 20}$$

Now solve this rational equation for r:

$$\begin{aligned} \frac{280}{r} = \frac{440}{r + 20} &\implies 280(r + 20) = 440r \\ &\implies 5600 = 160r \\ &\implies r = 35 \text{ mph} \end{aligned}$$

21. Let t represent the time it takes for each part of the trip, and let r represent the speed of the boat in still water. Then the actual speed of the boat going downstream is $r + 2$, and the speed upstream is $r - 2$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\text{Trip downstream: } 5 = (r + 2)t$$
$$\text{Trip upstream: } 2 = (r - 2)t$$

Solving for t in each equation, it follows that

$$\frac{5}{r + 2} = t = \frac{2}{r - 2}$$

Now solve this rational equation for r:

$$\begin{aligned} \frac{5}{r + 2} = \frac{2}{r - 2} &\implies 5(r - 2) = 2(r + 2) \\ &\implies 3r = 14 \\ &\implies r = \frac{14}{3} \text{ mph} \end{aligned}$$

23. Let t represent the time it takes for each part of the trip, and let c represent the speed of the current. Then the actual speed of the boat going downstream is $6+c$, and the speed upstream is $6-c$. Using the relationship

$$\text{distance} = \text{rate} \cdot \text{time},$$

this leads to the following two equations:

$$\begin{aligned} \text{Trip downstream: } & 10 = (6+c)t \\ \text{Trip upstream: } & 5 = (6-c)t \end{aligned}$$

Solving for t in each equation, it follows that

$$\frac{10}{6+c} = t = \frac{5}{6-c}$$

Now solve this rational equation for r:

$$\begin{aligned} \frac{10}{6+c} = \frac{5}{6-c} &\implies 10(6-c) = 5(6+c) \\ &\implies 30 = 15c \\ &\implies c = 2 \text{ mph} \end{aligned}$$

25. Let t represent the unknown time, r represent Jean's rate (reports/hour), and s represent Sanjay's rate. Using the relationship

$$\text{work} = \text{rate} \cdot \text{time},$$

this leads to the following three equations:

$$\begin{aligned} \text{Jean: } & 1 = r(t+15) \\ \text{Sanjay: } & 1 = st \\ \text{together: } & 1 = (r+s)\,10 \end{aligned}$$

Solve for r and s in the first two equations, and then plug those values into the third equation to obtain

$$1 = \left(\frac{1}{t+15} + \frac{1}{t}\right) 10$$

Now solve this rational equation for t:

$$\begin{aligned} 1 = \left(\frac{1}{t+15} + \frac{1}{t}\right) 10 &\implies t(t+15) = 10t + 10(t+15) \\ &\implies t^2 + 15t = 20t + 150 \\ &\implies t^2 - 5t - 150 = 0 \\ &\implies (t-15)(t+10) = 0 \\ &\implies t = 15 \text{ hours} \end{aligned}$$

27. Let t represent the unknown time, r represent Amelie's rate (reports/hour), and s represent Jean's rate. Using the relationship

$$\text{work} = \text{rate} \cdot \text{time},$$

this leads to the following three equations:

$$\begin{aligned}\text{Amelie: } 1 &= r(t+18)\\ \text{Jean: } 1 &= st\\ \text{together: } 1 &= (r+s)\,12\end{aligned}$$

Solve for r and s in the first two equations, and then plug those values into the third equation to obtain

$$1 = \left(\frac{1}{t+18} + \frac{1}{t}\right) 12$$

Now solve this rational equation for t:

$$\begin{aligned}1 = \left(\frac{1}{t+18} + \frac{1}{t}\right) 12 &\implies t(t+18) = 12t + 12(t+18)\\ &\implies t^2 + 18t = 24t + 216\\ &\implies t^2 - 6t - 216 = 0\\ &\implies (t-18)(t+12) = 0\\ &\implies t = 18 \text{ hours}\end{aligned}$$

29. Let t represent the unknown time, r represent Ricardo's rate (reports/hour), and s represent Sanjay's rate. Using the relationship

$$\text{work} = \text{rate} \cdot \text{time},$$

this leads to the following three equations:

$$\begin{aligned}\text{Ricardo: } 1 &= r(t+12)\\ \text{Sanjay: } 1 &= st\\ \text{together: } 1 &= (r+s)\,8\end{aligned}$$

Solve for r and s in the first two equations, and then plug those values into the third equation to obtain

$$1 = \left(\frac{1}{t+12} + \frac{1}{t}\right) 8$$

Now solve this rational equation for t:

$$\begin{aligned} 1 = \left(\frac{1}{t+12} + \frac{1}{t}\right) 8 &\implies t(t+12) = 8t + 8(t+12) \\ &\implies t^2 + 12t = 16t + 96 \\ &\implies t^2 - 4t - 96 = 0 \\ &\implies (t-12)(t+8) = 0 \\ &\implies t = 12 \text{ hours} \end{aligned}$$

31. Let t represent the unknown time, r represent Jean's rate (rooms/hour), and s represent Sanjay's rate. Using the relationship

$$\text{work} = \text{rate} \cdot \text{time},$$

this leads to the following three equations:

$$\begin{aligned} \text{Jean: } & 1 = 4r \\ \text{Sanjay: } & 1 = 7s \\ \text{together: } & 1 = (r+s)\,t \end{aligned}$$

Solve for r and s in the first two equations, and then plug those values into the third equation to obtain

$$1 = \left(\frac{1}{4} + \frac{1}{7}\right) t$$

Now solve this rational equation for t:

$$1 = \left(\frac{1}{4} + \frac{1}{7}\right) t \implies 1 = \left(\frac{11}{28}\right) t \implies t = \frac{28}{11} \text{ hours}$$