7.8 Applications of Rational Functions

In this section, we will investigate the use of rational functions in several applications.

**Number Problems**

We start by recalling the definition of the reciprocal of a number.

**Definition 1.** For any nonzero real number \( a \), the reciprocal of \( a \) is the number \( \frac{1}{a} \). Note that the product of a number and its reciprocal is always equal to the number 1. That is,

\[
a \cdot \frac{1}{a} = 1.
\]

For example, the reciprocal of the number 3 is \( \frac{1}{3} \). Note that we simply “invert” the number 3 to obtain its reciprocal \( \frac{1}{3} \). Further, note that the product of 3 and its reciprocal \( \frac{1}{3} \) is

\[
3 \cdot \frac{1}{3} = 1.
\]

As a second example, to find the reciprocal of \( -\frac{3}{5} \), we could make the calculation

\[
\frac{1}{-\frac{3}{5}} = 1 \div \left(-\frac{3}{5}\right) = 1 \cdot \left(-\frac{5}{3}\right) = -\frac{5}{3},
\]

but it’s probably faster to simply “invert” \( -\frac{3}{5} \) to obtain its reciprocal \( -\frac{5}{3} \). Again, note that the product of \( -\frac{3}{5} \) and its reciprocal \( -\frac{5}{3} \) is

\[
\left(-\frac{3}{5}\right) \cdot \left(-\frac{5}{3}\right) = 1.
\]

Let’s look at some applications that involve the reciprocals of numbers.

**Example 2.** The sum of a number and its reciprocal is \( \frac{29}{10} \). Find the number(s).

Let \( x \) represent a nonzero number. The reciprocal of \( x \) is \( \frac{1}{x} \). Hence, the sum of \( x \) and its reciprocal is represented by the rational expression \( x + \frac{1}{x} \). Set this equal to \( \frac{29}{10} \).

\[
x + \frac{1}{x} = \frac{29}{10}
\]

To clear fractions from this equation, multiply both sides by the common denominator 10x.

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1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
This equation is nonlinear (it has a power of \( x \) larger than 1), so make one side equal to zero by subtracting \( 29x \) from both sides of the equation.

\[
10x^2 - 29x + 10 = 0
\]

Let’s try to use the \( ac \)-test to factor. Note that \( ac = (10)(10) = 100 \). The integer pair \( \{-4, -25\} \) has product 100 and sum \(-29\). Break up the middle term of the quadratic trinomial using this pair, then factor by grouping.

\[
10x^2 - 4x - 25x + 10 = 0
\]

\[
2x(5x - 2) - 5(5x - 2) = 0
\]

\[
(2x - 5)(5x - 2) = 0
\]

Using the zero product property, either

\[
2x - 5 = 0 \quad \text{or} \quad 5x - 2 = 0.
\]

Each of these linear equations is easily solved.

\[
x = \frac{5}{2} \quad \text{or} \quad x = \frac{2}{5}
\]

Hence, we have two solutions for \( x \). However, they both lead to the same number-reciprocal pair. That is, if \( x = 5/2 \), then its reciprocal is \( 2/5 \). On the other hand, if \( x = 2/5 \), then its reciprocal is \( 5/2 \).

Let’s check our solution by taking the sum of the solution and its reciprocal. Note that

\[
\frac{5}{2} + \frac{2}{5} = \frac{25}{10} + \frac{4}{10} = \frac{29}{10},
\]

as required by the problem statement.

Let’s look at another application of the reciprocal concept.

**Example 3.** There are two numbers. The second number is 1 larger than twice the first number. The sum of the reciprocals of the two numbers is \( 7/10 \). Find the two numbers.

Let \( x \) represent the first number. If the second number is 1 larger than twice the first number, then the second number can be represented by the expression \( 2x + 1 \).

Thus, our two numbers are \( x \) and \( 2x + 1 \). Their reciprocals, respectively, are \( 1/x \) and \( 1/(2x + 1) \). Therefore, the sum of their reciprocals can be represented by the rational expression \( 1/x + 1/(2x + 1) \). Set this equal to \( 7/10 \).
\[
\frac{1}{x} + \frac{1}{2x+1} = \frac{7}{10}
\]

Multiply both sides of this equation by the common denominator \(10x(2x+1)\).

\[
10x(2x+1) \left(\frac{1}{x} + \frac{1}{2x+1}\right) = \left[\frac{7}{10}\right] 10x(2x+1)
\]

\[
10(2x+1) + 10x = 7x(2x + 1)
\]

Expand and simplify each side of this result.

\[
20x + 10 + 10x = 14x^2 + 7x
\]

\[
30x + 10 = 14x^2 + 7x
\]

Again, this equation is nonlinear. We will move everything to the right-hand side of this equation. Subtract \(30x\) and \(10\) from both sides of the equation to obtain

\[
0 = 14x^2 + 7x - 30x - 10
\]

\[
0 = 14x^2 - 23x - 10.
\]

Note that the right-hand side of this equation is quadratic with \(ac = (14)(-10) = -140\). The integer pair \(\{5, -28\}\) has product \(-140\) and sum \(-23\). Break up the middle term using this pair and factor by grouping.

\[
0 = 14x^2 + 5x - 28x - 10
\]

\[
0 = x(14x + 5) - 2(14x + 5)
\]

\[
0 = (x - 2)(14x + 5)
\]

Using the zero product property, either

\[
x - 2 = 0 \quad \text{or} \quad 14x + 5 = 0.
\]

These linear equations are easily solved for \(x\), providing

\[
x = 2 \quad \text{or} \quad x = -\frac{5}{14}.
\]

We still need to answer the question, which was to find two numbers such that the sum of their reciprocals is \(7/10\). Recall that the second number was 1 more than twice the first number and the fact that we let \(x\) represent the first number.

Consequently, if the first number is \(x = 2\), then the second number is \(2x + 1\), or \(2(2) + 1\). That is, the second number is 5. Let’s check to see if the pair \(\{2, 5\}\) is a solution by computing the sum of the reciprocals of 2 and 5.

\[
\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}
\]

Thus, the pair \(\{2, 5\}\) is a solution.
However, we found a second value for the first number, namely \(x = -5/14\). If this is the first number, then the second number is
\[
2 \left( -\frac{5}{14} \right) + 1 = -\frac{5}{7} + \frac{7}{7} = \frac{2}{7}.
\]
Thus, we have a second pair \(\{-5/14, 2/7\}\), but what is the sum of the reciprocals of these two numbers? The reciprocals are \(-14/5\) and \(7/2\), and their sum is
\[
-\frac{14}{5} + \frac{7}{2} = -\frac{28}{10} + \frac{35}{10} = \frac{7}{10},
\]
as required by the problem statement. Hence, the pair \(\{-14/5, 7/2\}\) is also a solution.

**Distance, Speed, and Time Problems**

When we developed the *Equations of Motion* in the chapter on quadratic functions, we showed that if an object moves with constant speed, then the distance traveled is given by the formula
\[
d = vt,
\]
where \(d\) represents the distance traveled, \(v\) represents the speed, and \(t\) represents the time of travel.

For example, if a car travels down a highway at a constant speed of 50 miles per hour (50 mi/h) for 4 hours (4 h), then it will travel
\[
d = vt
\]
\[
d = 50 \frac{\text{mi}}{\text{h}} \times 4 \text{ h}
\]
\[
d = 200 \text{ mi}.
\]
Let’s put this relation to use in some applications.

**Example 5.** A boat travels at a constant speed of 3 miles per hour in still water. In a river with unknown current, it takes the boat twice as long to travel 60 miles upstream (against the current) than it takes for the 60 mile return trip (with the current). What is the speed of the current in the river?

The speed of the boat in still water is 3 miles per hour. When the boat travels upstream, the current is against the direction the boat is traveling and works to reduce the actual speed of the boat. When the boat travels downstream, then the actual speed of the boat is its speed in still water increased by the speed of the current. If we let \(c\) represent the speed of the current in the river, then the boat’s speed upstream (against the current) is \(3 - c\), while the boat’s speed downstream (with the current) is \(3 + c\). Let’s summarize what we know in a distance-speed-time table (see Table 1).
Section 7.8 Applications of Rational Functions

Table 1. A distance, speed, and time table.

<table>
<thead>
<tr>
<th></th>
<th>$d$ (mi)</th>
<th>$v$ (mi/h)</th>
<th>$t$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>60</td>
<td>$3 - c$</td>
<td>?</td>
</tr>
<tr>
<td>Downstream</td>
<td>60</td>
<td>$3 + c$</td>
<td>?</td>
</tr>
</tbody>
</table>

Here is a useful piece of advice regarding distance, speed, and time tables.

**Distance, Speed, and Time Tables.** Because distance, speed, and time are related by the equation $d = vt$, whenever you have two boxes in a row of the table completed, the third box in that row can be calculated by means of the formula $d = vt$.

Note that each row of Table 1 has two entries entered. The third entry in each row is time. Solve the equation $d = vt$ for $t$ to obtain

$$t = \frac{d}{v}.$$  

The relation $t = d/v$ can be used to compute the time entry in each row of Table 1.

For example, in the first row, $d = 60$ miles and $v = 3 - c$ miles per hour. Therefore, the time of travel is

$$t = \frac{d}{v} = \frac{60}{3 - c}.$$  

Note how we’ve filled in this entry in Table 2. In similar fashion, the time to travel downstream is calculated with

$$t = \frac{d}{v} = \frac{60}{3 + c}.$$  

We’ve also added this entry to the time column in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>$d$ (mi)</th>
<th>$v$ (mi/h)</th>
<th>$t$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>60</td>
<td>$3 - c$</td>
<td>$\frac{60}{3 - c}$</td>
</tr>
<tr>
<td>Downstream</td>
<td>60</td>
<td>$3 + c$</td>
<td>$\frac{60}{3 + c}$</td>
</tr>
</tbody>
</table>

Table 2. Calculating the time column entries.

To set up an equation, we need to use the fact that the time to travel upstream is twice the time to travel downstream. This leads to the result

$$\frac{60}{3 - c} = 2 \left( \frac{60}{3 + c} \right),$$

or equivalently,
\[
\frac{60}{3-c} = \frac{120}{3+c}.
\]

Multiply both sides by the common denominator, in this case, \((3-c)(3+c)\).

\[
(3-c)(3+c) \left[ \frac{60}{3-c} \right] = \left[ \frac{120}{3+c} \right] (3-c)(3+c)
\]

\[
60(3+c) = 120(3-c)
\]

Expand each side of this equation.

\[
180 + 60c = 360 - 120c
\]

This equation is linear (no power of \(c\) other than 1). Hence, we want to isolate all terms containing \(c\) on one side of the equation. We add 120c to both sides of the equation, then subtract 180 from both sides of the equation.

\[
60c + 120c = 360 - 180
\]

From here, it is simple to solve for \(c\).

\[
180c = 180
\]

\[
c = 1.
\]

Hence, the speed of the current is 1 mile per hour.

It is important to check that the solution satisfies the constraints of the problem statement.

- If the speed of the boat in still water is 3 miles per hour and the speed of the current is 1 mile per hour, then the speed of the boat upstream (against the current) will be 2 miles per hour. It will take 30 hours to travel 60 miles at this rate.
- The speed of the boat as it goes downstream (with the current) will be 4 miles per hour. It will take 15 hours to travel 60 miles at this rate.

Note that the time to travel upstream (30 hours) is twice the time to travel downstream (15 hours), so our solution is correct.

Let’s look at another example.

**Example 6.** A speedboat can travel 32 miles per hour in still water. It travels 150 miles upstream against the current then returns to the starting location. The total time of the trip is 10 hours. What is the speed of the current?

Let \(c\) represent the speed of the current. Going upstream, the boat struggles against the current, so its net speed is \(32-c\) miles per hour. On the return trip, the boat benefits from the current, so its net speed on the return trip is \(32+c\) miles per hour. The trip each way is 150 miles. We’ve entered this data in Table 3.
Section 7.8 Applications of Rational Functions

Table 3. Entering the given data in a distance, speed, and time table.

<table>
<thead>
<tr>
<th></th>
<th>d (mi)</th>
<th>v (mi/h)</th>
<th>t (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>150</td>
<td>32 - c</td>
<td>?</td>
</tr>
<tr>
<td>Downstream</td>
<td>150</td>
<td>32 + c</td>
<td>?</td>
</tr>
</tbody>
</table>

Solving \( d = vt \) for the time \( t \),

\[
 t = \frac{d}{v}.
\]

In the first row of Table 3, we have \( d = 150 \) miles and \( v = 32 - c \) miles per hour.
Hence, the time it takes the boat to go upstream is given by

\[
 t = \frac{d}{v} = \frac{150}{32 - c}.
\]

Similarly, upon examining the data in the second row of Table 3, the time it takes the boat to return downstream to its starting location is

\[
 t = \frac{d}{v} = \frac{150}{32 + c}.
\]

These results are entered in Table 4.

Table 4. Calculating the time to go upstream and return.

<table>
<thead>
<tr>
<th></th>
<th>d (mi)</th>
<th>v (mi/h)</th>
<th>t (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>150</td>
<td>32 - c</td>
<td>150/(32 - c)</td>
</tr>
<tr>
<td>Downstream</td>
<td>150</td>
<td>32 + c</td>
<td>150/(32 + c)</td>
</tr>
</tbody>
</table>

Because the total time to go upstream and return is 10 hours, we can write

\[
 \frac{150}{32 - c} + \frac{150}{32 + c} = 10.
\]

Multiply both sides by the common denominator \((32 - c)(32 + c)\).

\[
 (32 - c)(32 + c) \left( \frac{150}{32 - c} + \frac{150}{32 + c} \right) = 10(32 - c)(32 + c)
\]

\[
 150(32 + c) + 150(32 - c) = 10(1024 - c^2)
\]

We can make the numbers a bit smaller by noting that both sides of the last equation are divisible by 10.

\[
 15(32 + c) + 15(32 - c) = 1024 - c^2
\]

Expand, simplify, make one side zero, then factor.
\[ 480 + 15c + 480 - 15c = 1024 - c^2 \]
\[ 960 = 1024 - c^2 \]
\[ 0 = 64 - c^2 \]
\[ 0 = (8 + c)(8 - c) \]

Using the zero product property, either
\[ 8 + c = 0 \quad \text{or} \quad 8 - c = 0, \]
providing two solutions for the current,
\[ c = -8 \quad \text{or} \quad c = 8. \]

Discarding the negative answer (speed is a positive quantity in this case), the speed of the current is 8 miles per hour.

Does our answer make sense?

- Because the speed of the current is 8 miles per hour, the boat travels 150 miles upstream at a net speed of 24 miles per hour. This will take \( \frac{150}{24} \) or 6.25 hours.
- The boat travels downstream 150 miles at a net speed of 40 miles per hour. This will take \( \frac{150}{40} \) or 3.75 hours.

Note that the total time to go upstream and return is 6.25 + 3.75, or 10 hours.

Let’s look at another class of problems.

**Work Problems**

A nice application of rational functions involves the amount of work a person (or team of persons) can do in a certain amount of time. We can handle these applications involving work in a manner similar to the method we used to solve distance, speed, and time problems. Here is the guiding principle.

**Work, Rate, and Time.** The amount of work done is equal to the product of the rate at which work is being done and the amount of time required to do the work. That is,

\[ \text{Work} = \text{Rate} \times \text{Time}. \]

For example, suppose that Emilia can mow lawns at a rate of 3 lawns per hour. After 6 hours,

\[ \text{Work} = 3 \frac{\text{lawns}}{\text{hr}} \times 6 \text{ hr} = 18 \text{ lawns}. \]

A second important concept is the fact that rates add. For example, if Emilia can mow lawns at a rate of 3 lawns per hour and Michele can mow the same lawns at a
rate of 2 lawns per hour, then together they can mow the lawns at a combined rate of 5 lawns per hour.

Let’s look at an example.

**Example 7.** Bill can finish a report in 2 hours. Maria can finish the same report in 4 hours. How long will it take them to finish the report if they work together?

A common misconception is that the times add in this case. That is, it takes Bill 2 hours to complete the report and it takes Maria 4 hours to complete the same report, so if Bill and Maria work together it will take 6 hours to complete the report. A little thought reveals that this result is nonsense. Clearly, if they work together, it will take them less time than it takes Bill to complete the report alone; that is, the combined time will surely be less than 2 hours.

However, as we saw above, the rates at which they are working will add. To take advantage of this fact, we set up what we know in a Work, Rate, and Time table (see Table 5).

- It takes Bill 2 hours to complete 1 report. This is reflected in the entries in the first row of Table 5.
- It takes Maria 4 hours to complete 1 report. This is reflected in the entries in the second row of Table 5.
- Let \( t \) represent the time it takes them to complete 1 report if they work together. This is reflected in the entries in the last row of Table 5.

<table>
<thead>
<tr>
<th></th>
<th>( w ) (reports)</th>
<th>( r ) (reports/h)</th>
<th>( t ) (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bill</td>
<td>1</td>
<td>?</td>
<td>2</td>
</tr>
<tr>
<td>Maria</td>
<td>1</td>
<td>?</td>
<td>4</td>
</tr>
<tr>
<td>Together</td>
<td>1</td>
<td>?</td>
<td>( t )</td>
</tr>
</tbody>
</table>

**Table 5.** A work, rate, and time table.

We have advice similar to that given for distance, speed, and time tables.

**Work, Rate, and Time Tables.** Because work, rate, and time are related by the equation

\[
\text{Work} = \text{Rate} \times \text{Time},
\]

whenever you have two boxes in a row completed, the third box in that row can be calculated by means of the relation \( \text{Work} = \text{Rate} \times \text{Time} \).

In the case of Table 5, we can calculate the rate at which Bill is working by solving the equation \( \text{Work} = \text{Rate} \times \text{Time} \) for the Rate, then substitute Bill’s data from row one of Table 5.

\[
\text{Rate} = \frac{\text{Work}}{\text{Time}} = \frac{1 \text{ report}}{2 \text{ h}}.
\]
Thus, Bill is working at a rate of \( \frac{1}{2} \) report per hour. Note how we’ve entered this result in the first row of Table 6. Similarly, Maria is working at a rate of \( \frac{1}{4} \) report per hour, which we’ve also entered in Table 6.

We’ve let \( t \) represent the time it takes them to write 1 report if they are working together (see Table 5), so the following calculation gives us the combined rate.

\[
\text{Rate} = \frac{\text{Work}}{\text{Time}} = \frac{1 \text{ report}}{t \text{ h}}.
\]

That is, together they work at a rate of \( \frac{1}{t} \) reports per hour. This result is also recorded in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>( w ) (reports)</th>
<th>( r ) (reports/h)</th>
<th>( t ) (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bill</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
<td>2</td>
</tr>
<tr>
<td>Maria</td>
<td>1</td>
<td>( \frac{1}{4} )</td>
<td>4</td>
</tr>
<tr>
<td>Together</td>
<td>1</td>
<td>( \frac{1}{t} )</td>
<td>( t )</td>
</tr>
</tbody>
</table>

Table 6. Calculating the Rate entries.

In our discussion above, we pointed out the fact that rates add. Thus, the equation we seek lies in the Rate column of Table 6. Bill is working at a rate of \( \frac{1}{2} \) report per hour and Maria is working at a rate of \( \frac{1}{4} \) report per hour. Therefore, their combined rate is \( \frac{1}{2} + \frac{1}{4} \) reports per hour. However, the last row of Table 6 indicates that the combined rate is also \( \frac{1}{t} \) reports per hour. Thus,

\[
\frac{1}{2} + \frac{1}{4} = \frac{1}{t}.
\]

Multiply both sides of this equation by the common denominator \( 4t \).

\[
(4t) \left[ \frac{1}{2} + \frac{1}{4} \right] = \left[ \frac{1}{t} \right] (4t)
\]

\[
2t + t = 4,
\]

This equation is linear (no power of \( t \) other than 1) and is easily solved.

\[
3t = 4
\]

\[
t = \frac{4}{3}
\]

Thus, it will take \( \frac{4}{3} \) of an hour to complete 1 report if Bill and Maria work together.

Again, it is very important that we check this result.

• We know that Bill does \( \frac{1}{2} \) reports per hour. In \( \frac{4}{3} \) of an hour, Bill will complete

\[
\text{Work} = \frac{1}{2} \text{ reports/h} \times \frac{4}{3} \text{ h} = \frac{2}{3} \text{ reports}.
\]

That is, Bill will complete \( \frac{2}{3} \) of a report.

• We know that Maria does \( \frac{1}{4} \) reports per hour. In \( \frac{4}{3} \) of an hour, Maria will complete
That is, Maria will complete 1/3 of a report.

Clearly, working together, Bill and Maria will complete 2/3 + 1/3 reports, that is, one full report.

Let’s look at another example.

**Example 8.** It takes Liya 7 more hours to paint a kitchen than it takes Hank to complete the same job. Together, they can complete the same job in 12 hours. How long does it take Hank to complete the job if he works alone?

Let $H$ represent the time it take Hank to complete the job of painting the kitchen when he works alone. Because it takes Liya 7 more hours than it takes Hank, let $H + 7$ represent the time it takes Liya to paint the kitchen when she works alone. This leads to the entries in Table 7.

<table>
<thead>
<tr>
<th></th>
<th>$w$ (kitchens)</th>
<th>$r$ (kitchens/h)</th>
<th>$t$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hank</td>
<td>1</td>
<td>?</td>
<td>$H$</td>
</tr>
<tr>
<td>Liya</td>
<td>1</td>
<td>?</td>
<td>$H + 7$</td>
</tr>
<tr>
<td>Together</td>
<td>1</td>
<td>?</td>
<td>12</td>
</tr>
</tbody>
</table>

**Table 7.** Entering the given data for Hank and Liya.

We can calculate the rate at which Hank is working alone by solving the equation $\text{Work} = \text{Rate} \times \text{Time}$ for the Rate, then substituting Hank’s data from row one of Table 7.

$$\text{Rate} = \frac{\text{Work}}{\text{Time}} = \frac{1 \text{ kitchen}}{H \text{ hour}}$$

Thus, Hank is working at a rate of $1/H$ kitchens per hour. Similarly, Liya is working at a rate of $1/(H + 7)$ kitchens per hour. Because it takes them 12 hours to complete the task when working together, their combined rate is $1/12$ kitchens per hour. Each of these rates is entered in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>$w$ (kitchens)</th>
<th>$r$ (kitchens/h)</th>
<th>$t$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hank</td>
<td>1</td>
<td>$1/H$</td>
<td>$H$</td>
</tr>
<tr>
<td>Liya</td>
<td>1</td>
<td>$1/(H + 7)$</td>
<td>$H + 7$</td>
</tr>
<tr>
<td>Together</td>
<td>1</td>
<td>$1/12$</td>
<td>12</td>
</tr>
</tbody>
</table>

**Table 8.** Calculating the rates.

Because the rates add, we can write

$$\frac{1}{H} + \frac{1}{H + 7} = \frac{1}{12}.$$
Multiply both sides of this equation by the common denominator $12H(H + 7)$.

$$12H(H + 7) \left( \frac{1}{H} + \frac{1}{H + 7} \right) = \left( \frac{1}{12} \right) 12H(H + 7)$$

$$12(H + 7) + 12H = H(H + 7)$$

Expand and simplify.

$$12H + 84 + 12H = H^2 + 7H$$

$$24H + 84 = H^2 + 7H$$

This last equation is nonlinear, so make one side zero by subtracting $24H$ and 84 from both sides of the equation.

$$0 = H^2 + 7H - 24H - 84$$

$$0 = H^2 - 17H - 84$$

Note that $ac = (1)(-84) = -84$. The integer pair $\{4, -21\}$ has product $-84$ and sums to $-17$. Hence,

$$0 = (H + 4)(H - 21).$$

Using the zero product property, either

$$H + 4 = 0 \quad \text{or} \quad H - 21 = 0,$$

leading to the solutions

$$H = -4 \quad \text{or} \quad H = 21.$$

We eliminate the solution $H = -4$ from consideration (it doesn’t take Hank negative time to paint the kitchen), so we conclude that it takes Hank 21 hours to paint the kitchen.

Does our solution make sense?

- It takes Hank 21 hours to complete the kitchen, so he is finishing $\frac{1}{21}$ of the kitchen per hour.
- It takes Liya 7 hours longer than Hank to complete the kitchen, namely 28 hours, so she is finishing $\frac{1}{28}$ of the kitchen per hour.

Together, they are working at a combined rate of

$$\frac{1}{21} + \frac{1}{28} = \frac{4}{84} + \frac{3}{84} = \frac{7}{84} = \frac{1}{12},$$

or $\frac{1}{12}$ of a kitchen per hour. This agrees with the combined rate in Table 8.